

Structural Theorems for Computation-Realization Geometry

Interaction, Safe Commitment, Network Embedding, Conservation, and Scaling

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Abstract

This paper develops selected exact deterministic fragments of Computation–Realization Geometry rather than claiming closure of the complete nonlinear, randomized, approximate, or physically embedded theory. It establishes circuit–protocol translations under named accounting rules, an exact interactive directional cut-rank orthant for finite-field linear computations, deterministic carrier/network-function-code equivalence, schedule-reachable temporal cut-state conservation, fractional certificate packing, hierarchy coherence, and exact retention statements for monotone and additive-price decision classes. The Safe-Commitment Law is supplemented by a strictly increasing coercive-cost separation result. Regular-variation, positive-system, boundary-collapse, and architecture–technology scaling theorems close additional declared slices. Each result keeps computation, ownership, legal transformations, hierarchy, accounting, and evidence class explicit, so theorems can be composed without transferring conclusions across incompatible contracts.

*Companion to **A Communication Theory of Computation Realization** (v3.2.0), DOI [10.5281/zenodo.22048090](https://doi.org/10.5281/zenodo.22048090). This manuscript develops selected exact deterministic fragments of the canonical theory: interaction, network embedding, conservation, hierarchy, and safe commitment.*

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1 Introduction

Computation-realization geometry represents a declared computation by its legal realization family, typed joint obligations, hierarchy maps, and decision-sufficient quotients before technology selects an embedding. The framework becomes a theory when achievability, converse, observation, retention, and scaling close on the same object.

This paper proves the principal closure theorems. For interactive finite-field linear computation on a tree, directional cross-cut ranks are simultaneous lower bounds and one quotient realization attains every edge bound. Directed carriers recover deterministic zero-error network function codes. Temporal cut-state certificates yield a native conservation law and a fractional packing theorem. Continuous monotone observations reconstruct the closed upper obligation region, and nonnegative linear prices reconstruct its convex body. The Safe-Commitment Law turns these reconstruction theorems into necessary-and-sufficient conditions for lossless reduction of a realization family.

The finite technology closure uses semialgebraic sign decomposition to retain complete tied labels across feasibility, bottleneck, witness, and architecture changes; argmin hitting gives the exact retained-family condition over a named technology domain. The scaling closure separates observable output class from generative mechanism: stable dilation limits and phase ratios characterize output, while projectively mixing positive cocycles and cap-response laws supply mechanisms and converses.

These results place communication-complexity converses, network regions, convex observation duality, parametric architecture phases, and multiscale positive dynamics in one computation-first quantifier order. The paper makes that dependency architecture explicit and identifies the exact statement closed at every layer.

2 Closure in neighboring theories

The closure results combine four mathematical traditions. Communication complexity supplies one-cut transcript and range converses [16, 3, 4]; network coding supplies carrier-load regions and the distinction between routing and legal local coding [6, 7, 5]. The exact finite-field orthant proved here strengthens a collection of separate cut bounds into one simultaneously attained directional lower corner on every edge of an arbitrary tree placement.

Convex and vector optimization supply upper images, separation, and support reconstruction [12, 17]. Blackwell comparison and abstract interpretation provide neighboring ideas of decision sufficiency and sound abstraction [18, 19]. The Safe-Commitment Law specializes these ideas to legal computation realizations: continuous monotone responses reconstruct Γ , nonnegative linear prices reconstruct $\mathcal{K}$, and equality of the required quotient is necessary and sufficient for universal retention in the declared class.

Regular variation and discrete scale invariance characterize stable output ratios [1, 2]; positive-operator theory supplies projective contraction and asymptotic mode selection [13, 14]. The closure theorem joins the output and mechanism levels: phase-ratio limits characterize the observable sequence, while an asymptotically periodic projectively mixing cocycle supplies a sufficient generative basin with explicit perturbation control.

Semialgebraic geometry supplies finite sign-cell decompositions [11]. Applied to a finite architecture registry, it yields complete set-valued technology phases, including feasibility, branch, bottleneck, witness, and architecture ties. Argmin-hitting retention then closes the finite-ledger decision problem. The common feature across all four traditions is exact closure relative to a declared object and observation class; CRG provides the computation-first object in which they compose.

Sequence and nonhomogeneous-product lineage. The sequence-level regular-variation results sit in the classical theory of regularly varying sequences [20, 21]. The positive-product mechanism is related to weak ergodicity and nonhomogeneous products of nonnegative matrices [22, 23, 24, 25].

3 Preliminaries and contract scope

A *terminalized computation* specifies finite input and output alphabets, input and output ownership, and the required map. A resource contract ρ fixes allowed local primitives, work, depth, storage, replication or recomputation, error, hierarchy rules, and communication accounting. A legal realization is compared only after these quantities are fixed.

Definition 3.1 (Terminalized deterministic computation). *A terminalized deterministic computation is a map*

$$F : \mathcal{X} = \prod_{i \in I} X_i \longrightarrow \mathcal{Y} = \prod_{j \in O} Y_j,$$

where every alphabet is finite. A hierarchy consists of a finite tree $\mathcal{T} = (V, E)$ and placement maps $\lambda_I : I \rightarrow V$ and $\lambda_O : O \rightarrow V$. Vertex v owns the local input alphabet

$$X_v = \prod_{i: \lambda_I(i)=v} X_i$$

and must produce the local output alphabet

$$Y_v = \prod_{j: \lambda_O(j)=v} Y_j.$$

Definition 3.2 (Core resource contract). A core deterministic resource contract is a tuple

$$\rho = (\mathcal{B}, S, D, M, R, \varepsilon, \Lambda, \chi),$$

where $\mathcal{B}$ is the allowed local primitive basis, S is a local-work or circuit-size budget, D a causal-depth budget, M a storage budget, R a replication or recomputation budget, ε an allowed error (zero in the exact results below), Λ the admissible hierarchy and terminal-placement rules, and χ the communication-accounting type. A ρ -legal realization P is a finite hierarchical protocol or circuit satisfying these typed bounds. Contract relaxation is semantic inclusion: $\rho \leq \rho'$ exactly when every ρ -legal realization is ρ' -legal.

To make hierarchy contraction formal, each scalar bound is represented by a finitely supported charge on hierarchy sites or interfaces together with a declared associative aggregation law. Local work and storage use addition; retained interface traffic and replication charges use addition over surviving interfaces; causal depth and protocol rounds use max-plus path composition; terminal placement uses direct image; exact error remains zero, while an approximate extension must declare an associative error-composition operation $\oplus_\varepsilon$; and χ is preserved. A connected-fiber contraction $q : \mathcal{T} \rightarrow \mathcal{T}'$ pushes a charge forward by aggregating all charges in each fiber and on each induced surviving interface.

A resource budget is a legality constraint on the complete realization; a resource charge is the additive or max-plus object pushed through a hierarchy map. A budget need not equal the sum of local charges unless the contract declares that aggregation rule. Thus, for example, a package-wide peak or shared memory pool is not counted once per stage merely because stage-local timing expressions are written separately. Extensions may add stochastic, analog, reversible, or technology-specific coordinates, but they must specify legal-realization semantics, an associative aggregation law, and pushforward behavior before the coherence results apply.

Proposition 3.3 (Functoriality of contract pushforward). For every core contract with the aggregation laws of Theorem 3.2, connected-fiber contractions satisfy

$$\text{id}_* \rho = \rho, \quad (r \circ q)_* \rho = r_*(q_* \rho).$$

Proof. For an additive local charge a , the charge assigned to a coarse site w by the composite contraction is

$$((r \circ q)_* a)(w) = \sum_{v: (r \circ q)(v)=w} a(v) = \sum_{u: r(u)=w} \sum_{v: q(v)=u} a(v) = (r_*(q_* a))(w).$$

The same regrouping proves the claim for additive interface and replication charges. Max-plus path aggregation is associative because path concatenation and max are associative; direct image of terminal placement is functorial; exact error is fixed at zero; and every admitted approximate coordinate is functorial by the required associativity of $\oplus_\varepsilon$. The primitive basis and accounting type are unchanged. Identity maps leave every support and charge unchanged, so componentwise equality gives both identities. $\square$

Definition 3.4 (Typed interface signature). For an oriented hierarchy interface e , a typed signature is

$$\mu_e = (s_e, \nu_e, t_e, d_e, a_e, \pi_e),$$

where s_e is bits per state or message, ν_e is message events per operation, $t_e = s_e \nu_e$ is bits per operation, d_e is a causal-round or depth contribution, a_e is an activity factor, and π_e is an optional traffic or burst distribution. An analysis may project to any declared subset. Static cut accounting uses s_e or a fixed-bit width; throughput and link calculations use t_e ; latency uses d_e ; and dynamic energy requires t_e together with a_e and a technology model. These coordinates are not interchangeable.

Definition 3.5 (Interface-accounting convention). *For a finite message alphabet Σ , define its semantic log-cardinality and fixed-length binary width by*

$$\ell_{\text{sem}}(\Sigma) = \log_2 |\Sigma|, \quad \ell_{\text{bit}}(\Sigma) = \lceil \log_2 |\Sigma| \rceil.$$

For a sequence of message events, capacities add event by event. The contract records $\chi \in \{\text{sem}, \text{bit}\}$, or an explicitly specified richer coding rule. Semantic capacity is exact for state-space and product identities; fixed-bit traffic is the physical convention for Boolean circuit nets. The two conventions coincide at one bit for a binary alphabet.

Unless stated otherwise, the event schedule is fixed independently of the input, conditional absence is charged as an alphabet symbol, and opposite directions are charged separately. The event sum is provisioned alphabet capacity, not Shannon entropy of a reachable-transcript distribution. Joint coding across events, expected variable length, compression, retransmission, broadcast cost, and link sharing require a richer χ stating decoder, timing, and charging rules. Bits per operation alone do not specify instantaneous bandwidth or latency; event-rate, burst, and service coordinates are also required.

4 Computation-realization geometry

Definition 4.1 (Admissible decomposition system). *The decomposition system $\mathbf{Dec}_\rho(F)$ has as objects the tree hierarchies, terminal placements, and local site rules admitted by Λ . Its structure maps are semantic hierarchy isomorphisms and connected-fiber contractions $q : \mathcal{T} \rightarrow \mathcal{T}'$ equipped with the typed pushforward $q_*\rho$ of Theorem 3.2. Identity and composition are inherited from the tree maps. A contract outside the core calculus is admissible only when its legal-realization semantics and pushforward coherence are supplied explicitly.*

Definition 4.2 (Realization bundle, signature regions, and CRG). *Fix F , ρ , and $\mathcal{T} \in \mathbf{Dec}_\rho(F)$. Let*

$$\mathcal{P}_F^\rho(\mathcal{T})$$

be the registry of ρ -legal hierarchical realizations P , modulo semantic-preserving renaming of internal states. Let

$$\sigma_{\mathcal{T}} : \mathcal{P}_F^\rho(\mathcal{T}) \longrightarrow \mathbb{R}_+^{d(\mathcal{T})}$$

extract the declared typed interface signature, or a declared projection of it. Define the exact attainable region

$$\mathcal{R}_F^\rho(\mathcal{T}) = \sigma_{\mathcal{T}}(\mathcal{P}_F^\rho(\mathcal{T})),$$

the closed upper obligation region

$$\Gamma_F^\rho(\mathcal{T}) = \text{cl}(\text{up } \mathcal{R}_F^\rho(\mathcal{T})),$$

and the convex linear-cost body

$$\mathcal{K}_F^\rho(\mathcal{T}) = \text{cl conv } \Gamma_F^\rho(\mathcal{T}).$$

The computation-realization geometry is the decomposition-indexed system

$$\mathfrak{G}_F^\rho = \left(\mathbf{Dec}_\rho(F), \left\{ \mathcal{P}_F^\rho(\mathcal{T}) \xrightarrow{\sigma_{\mathcal{T}}} \mathcal{R}_F^\rho(\mathcal{T}), \Gamma_F^\rho(\mathcal{T}), \mathcal{K}_F^\rho(\mathcal{T}) \right\}_{\mathcal{T}}, \{\pi_q\}_q \right).$$

The levels retain different information. The registry $\mathcal{P}$ distinguishes architectures, event structures, and embeddings. The exact image $\mathcal{R}$ retains jointly attainable, possibly nonconvex signatures but forgets architecture identity inside a signature fiber. The upper region Γ is the exact quotient for continuous monotone interface-obligation observations. The convex upper body $\mathcal{K}$ is the further quotient seen by additive nonnegative prices and explicitly admitted amortized mixtures. The term *geometry* denotes this resource-indexed feasible system, not a Euclidean placement, signal constellation, or smooth manifold. A selected algorithm may have an intermodule communication geometry in the classical sense [15]; here topology and technology remain unfixed until the contract and observation are chosen.

The INDEX interaction boundary of Theorem 4.3 gives a concrete two-coordinate slice. For that declared two-party contract, $\mathcal{R}$ contains the one-way signature $(n, 0)$ and the interactive signature $(1, \lceil \log_2 n \rceil)$ in the two directed-traffic coordinates. The region Γ adds provisioned points that dominate an attainable signature, while $\mathcal{K}$ further retains the convex information visible to additive nonnegative prices. Thus $\mathcal{P}$ remembers the protocols, $\mathcal{R}$ their jointly attainable signatures, Γ the monotone obligations, and $\mathcal{K}$ the linear-price quotient.

4.1 Where interaction changes the signature

Example 4.3 (INDEX interaction boundary). *Alice holds $x \in \{0,1\}^n$, Bob holds $i \in [n]$, and Bob must output x_i . A one-way Alice-to-Bob summary must distinguish all 2^n strings, so its semantic capacity is n bits. A two-round protocol lets Bob send i using $\lceil \log_2 n \rceil$ bits and Alice return x_i using one bit. Thus*

$$D_{\text{one-way}}^{A \rightarrow B}(\text{INDEX}_n) = n, \quad D_{\text{interactive}}(\text{INDEX}_n) \leq \lceil \log_2 n \rceil + 1.$$

The legal directional signatures $(n, 0)$ and $(1, \lceil \log_2 n \rceil)$ can be selected by different price vectors. With uniform bit prices and $n \geq 4$, the interactive realization wins; if Bob-to-Alice traffic is sufficiently expensive, the one-way realization wins. A one-way contextual summary, total communication, and a technology-weighted interface objective are different observations.

In the noninteractive model, contextual quotients are both constructive points and matching lower bounds. In a general model, a semantic outer bound requires a problem-specific converse from communication complexity, information complexity, algebra, or another appropriate method. The operational discipline is therefore to retain certified brackets

$$\mathcal{R}_{F,\text{in}}^\rho(\mathcal{T}) \subseteq \mathcal{R}_F^\rho(\mathcal{T}) \subseteq \mathcal{R}_{F,\text{out}}^\rho(\mathcal{T})$$

and declare every gap.

4.2 Complete interactive finite-field linear orthant

Theorem 4.4 (Linear contextual states are cut ranks). *Let $F(x) = Mx$ over $\text{GF}(2)$, split inputs as $x = (x_A, x_B)$, and place a set of outputs Y_B on side B . If $M_{B,A}$ is the submatrix mapping x_A to outputs required on B , then $a, a' \in \text{GF}(2)^A$ are contextually equivalent for $A \rightarrow B$ exactly when*

$$M_{B,A}(a - a') = 0.$$

The quotient has $2^{\text{rank } M_{B,A}}$ classes, and

$$\boxed{\kappa_{A \rightarrow B} = \text{rank}_{\text{GF}(2)} M_{B,A}.}$$

Proof. For any outside assignment b , the outputs on B differ by $M_{B,A}(a - a')$. They agree for every b exactly when this vector is zero. The number of cosets of $\ker M_{B,A}$ is $2^{\text{rank } M_{B,A}}$. $\square$

Lemma 4.5 (Interactive directional range bound). *Delete one tree edge and call the sides A and B . In any exact deterministic finite causal protocol with a static charged event schedule,*

$$t_{A \rightarrow B} \geq \max_b \log_2 |\{F_B(a, b) : a \in X_A\}|$$

under semantic alphabet accounting; fixed binary width rounds the right side up.

Proof. Fix b and retain the ordered tuple of every A -to- B symbol. Given that tuple and the receiver input, every receiver-local event and reverse message is determined recursively by induction over the event DAG. Equal forward tuples must therefore give equal required receiver outputs. The tuple has at least the displayed number of values and at most the product of its event alphabet cardinalities. $\square$

Lemma 4.6 (Local composition of linear quotient states). *For a component C exposed by deleting a tree edge, let*

$$L_C = M_{O_C, I_C}, \quad Q_C = \mathbb{F}_q^{I_C} / \ker L_C,$$

where q is the field cardinality. If $O_B \subseteq O_C$, then

$$\boxed{\ker M_{O_C, I_C} \subseteq \ker M_{O_B, I_C}.}$$

Consequently $M_{O_B, I_C} x_C$ is a well-defined linear function of Q_C . If disjoint incident components $C_1, \dots, C_s$ meet at v and O_B lies outside every C_i , their aggregate contribution to O_B is a linear function of x_{I_v} and the states Q_{C_i} .

Proof. Deleting output rows from $M_{O_{\bar{C}}, I_C}$ can only enlarge its kernel. Thus

$$L_C x_C \mapsto M_{O_B, I_C} x_C$$

is well defined on $\text{im } L_C \cong Q_C$. Apply this factor map to each incident component and add the resulting vectors together with the local term $M_{O_B, I_v} x_{I_v}$. $\square$

Definition 4.7 (Interactive linear traffic contract). *Let $F(x) = Mx$ over a fixed finite field $\mathbb{F}_q$, with input coordinates and required output rows placed on $\mathcal{T}$. For an oriented edge $e = A \rightarrow B$, put*

$$M_e = M_{O_B, I_A}, \quad r_e = \text{rank}_{\mathbb{F}_q} M_e.$$

The contract admits exact deterministic finite causal tree protocols, arbitrary deterministic local maps (not necessarily linear), and a static charged event schedule. Each vertex begins with only its placed input and fixed input-independent advice; no input-dependent state is prepositioned across a cut. Traffic is charged by Theorem 3.5; local work, storage, rounds, and depth are unrestricted in this slice. Conditional absence remains an ordinary charged symbol.

Theorem 4.8 (Exact interactive linear lower corner and orthant). *Under Theorem 4.7, arbitrary interaction and nonlinear local encoding cannot reduce the directional load below $r_e \log_2 q$. One quotient realization attains all edge bounds simultaneously. Hence*

$$\Gamma_F^\rho(\mathcal{T}) = \mathcal{K}_F^\rho(\mathcal{T}) = \prod_{e \in \vec{E}(\mathcal{T})} [r_e \log_2 q, \infty)$$

under semantic traffic, with lower corner $(\lceil r_e \log_2 q \rceil)_e$ under fixed binary widths. For every nonnegative directional price vector w ,

$$\inf_P \langle w, t(P) \rangle = \sum_{e \in \vec{E}(\mathcal{T})} w_e r_e \log_2 q.$$

Proof. Fix the complete B -side input. The required B -side output ranges over an affine translate of $\text{im } M_{O_B, I_A}$, which has q^{r_e} points. The directional range bound gives the converse without assuming linear encoders. Contextual equivalence identifies the sender quotient

$$\mathbb{F}_q^{I_A} / \ker M_{O_B, I_A},$$

of dimension r_e .

For simultaneous achievability, root the tree arbitrarily. In the inward pass, suppose child components C_i meet their parent at v and $C = \{v\} \cup_i C_i$. Since $O_{\bar{C}} \subseteq O_{\bar{C}_i}$, Theorem 4.6 lets v compute Q_C from its local input and the child states. In the outward pass toward child C_j , the parent combines its local input, the inward states of all sibling components, and its incoming outward state. The target rows O_{C_j} lie outside every contributing component, so the same lemma computes the complementary state sent into C_j . Every vertex then combines its local input and incident component states to produce its placed outputs. Identifying each quotient with $\text{im } L_C$ and choosing a basis sends exactly r_e field symbols on every oriented edge in one realization. The attainable lower corner and componentwise converse give the upper orthant, already convex. $\square$

This theorem closes the static charged traffic region, not the regions for rounds, depth, local work, storage, approximation, randomization, variable-length expected cost, or graph carriers. INDEX lies outside this linear class because its opposite input selects which distinction matters; for a linear map the cross-side image is context independent. On a graph with parallel paths, the rank converse applies to aggregate cut traffic, not independently to each arc. The one-cut transcript and rank facts are standard; the joint conclusion is the simultaneous, arbitrary-placement directed orthant even when reverse interaction is free and local encoders may be nonlinear [3, 4, 5].

4.3 Carrier graphs and network computation

A tree records nested accounting boundaries; a directed carrier graph records which local channels exist. Let $\mathcal{N} = (\mathcal{G}_{\text{car}}, \alpha_I, \alpha_O)$ be a finite directed acyclic multigraph with source-access and output-demand relations. A carrier realization assigns a finite alphabet to every arc, a local encoder from the tail's accessible sources and incoming symbols, and exact decoders at demand vertices. Its signature is the vector $\ell(P)$ of log alphabet sizes. Define the equal-instance normalized zero-error link-load region

$$\mathcal{L}_F(\mathcal{N}) = \text{cl up } \bigcup_{k \geq 1} \left\{ \frac{\ell(P)}{k} : P \text{ realizes } F^{\otimes k} \text{ exactly on } \mathcal{N} \right\}.$$

Block concatenation makes this closed upper region convex.

Theorem 4.9 (Carrier/network-code equivalence). *Under deterministic zero-error semantics on a finite acyclic carrier, single-shot carrier realizations and network function codes are identical up to internal-symbol renaming, with identical arc loads. Applying the bijection to $F^{\otimes k}$ for every k , normalizing by k , and taking closed upper block closure gives exactly the zero-error network-computation link-load region. Identity or multicast output demands specialize this to ordinary multi-source network codes under equal replication of the source instance fixed by F . This does not identify the full independent-source rate-vector region.*

Proof. Both objects assign precisely the same local encoder to each directed arc and the same decoder to each demand vertex. A topological order makes every carrier encoder a legal network-code encoder; the reverse translation changes nothing. The identity translation commutes with independent block products, normalization, and closure. $\square$

For arc capacities c_a , the asymptotic computation rate is

$$\sup(\{0\} \cup \{R > 0 : \exists \ell \in \mathcal{L}_F(\mathcal{N}), R\ell_a \leq c_a \forall a\}).$$

The adjoined zero fixes the gauge: if the carrier is zero-error unsolvable and $\mathcal{L}_F(\mathcal{N}) = \emptyset$, its computation capacity is zero. Placing carrier vertices in $\mathcal{T}$ linearly projects arc loads to tree-cut loads, but parallel arcs show the projection is not injective. Matched network-code solvability and nonlinear-code counterexamples therefore transfer exactly. Linear-versus-nonlinear comparisons require the explicit subcontract in which source and arc alphabets are vector spaces over one fixed field and all local maps are linear.

The inherited undecidability statement is specifically the variable-alphabet matched-solvability problem: given finite source, carrier, and demand incidence, decide whether some common alphabet size at least two and deterministic local encoders satisfy every demand exactly. It does not make fixed-alphabet feasibility or every network-code decision problem undecidable; a finite instance with every alphabet supplied is decidable by finite enumeration [6, 7, 8].

4.4 Temporal cut-state conservation and scoped projections

Definition 4.10 (Schedule-reachable temporal cut-state certificate). *Fix a deterministic realization with a finite acyclic event DAG, a static charged schedule, a complete site cut $A \mid B$, and a service window I . Conditional absence is an ordinary charged symbol. With receiver-local exogenous context fixed, let S_I be the receiver's retained state at the start of the window, with $\log_2 |S_I| \leq m_I$; let $\mathcal{A}_I$ contain all forward A -to- B events in the window, with $\ell_a = \log_2 |\Sigma_a|$; and let O_I be a receiver state or output required at the end, with*

$$\kappa_I = \log_2 |\text{im } O_I|$$

as compatible sender histories vary. The certificate is valid when: (i) every retained-state coordinate is available at the beginning of I ; (ii) every charged incoming event occurs before the certified output and lies on a directed path from a source-side input or state event to O_I ; (iii) every directed dependency path by which a varied source-side history can alter O_I intersects either the retained-state set or the charged incoming-event set; and (iv) each charged event is counted once under the registered accounting rule. These conditions are decidable by reachability and cut tests on the finite event DAG.

Theorem 4.11 (Cut-state conservation). *Every temporal cut-state certificate satisfies*

$$\boxed{\kappa_I \leq m_I + \sum_{a \in \mathcal{A}_I} \ell_a}, \quad \boxed{\sum_{a \in \mathcal{A}_I} \ell_a \geq (\kappa_I - m_I)_+}.$$

Interaction and nonlinear local encoding do not improve the inequality.

Proof. The initial receiver state and ordered forward transcript determine every receiver-local event, reverse message, and required output by induction over the event DAG. Their joint alphabet has at least 2^{κ_I} values and at most $2^{m_I} \prod_{a \in \mathcal{A}_I} 2^{\ell_a}$. Taking logarithms proves both forms. $\square$

Theorem 4.12 (Fractional certificate packing). *For any family of valid certificates j , with forward-event sets $\mathcal{A}_j$ and fractional weights $\alpha_j \geq 0$ satisfying*

$$\sum_{j:a \in \mathcal{A}_j} \alpha_j \leq 1 \quad \text{for every event } a,$$

total charged traffic obeys

$$Q \geq \sum_j \alpha_j (\kappa_j - m_j)_+.$$

Proof. This is the standard fractional-packing/linear-programming composition [30]. Multiply each instance of Theorem 4.11 by α_j , sum, exchange the order of summation, and use the per-event charging constraint. $\square$

Three classical projections require three additional, explicitly distinct contracts.

- (i) *Bandwidth–time.* If a complete cut supplies at most B bits per unit time and no relevant state is prepositioned, cut-state conservation gives

$$BT \geq \kappa.$$

- (ii) *Atomic I/O partition.* Under the standard atomic-word red–blue pebble contract, let $H_D(S)$ be the minimum number of parts in a classical S -partition of the noninput vertices of DAG D . The parts are disjoint and exhaustive, their quotient is acyclic, each has a dominator set of at most S vertices intercepting every input-to-part path, and

$$\text{Min}(V_i) = \{v \in V_i : \text{no immediate successor of } v \text{ lies in } V_i\}$$

has cardinality at most S . Segmenting an execution after every M I/O moves creates a Hong–Kung $2M$ -partition, hence

$$Q \geq M(H_D(2M) - 1).$$

This inequality follows from the atomic-DAG partition spectrum and pebble contract; it is not obtained by silently identifying a memory move with a static tree-cut symbol.

- (iii) *Two-dimensional area–time.* Suppose every admissible balanced bisection has an orientation $A_0 \rightarrow B_0$ with forward service $B_{\text{bis}}^{A_0 \rightarrow B_0} \leq \gamma \sqrt{A}$, a valid oriented cut-state certificate requiring κ_{bis} distinctions, and at most m_{bis} relevant prepositioned bits on B_0 . With

$$\kappa_{\text{bis}}^{\text{net}} = (\kappa_{\text{bis}} - m_{\text{bis}})_+,$$

conservation yields

$$AT^2 \geq \frac{(\kappa_{\text{bis}}^{\text{net}})^2}{\gamma^2}.$$

This projection uses cut-state conservation plus the two-dimensional service and separator law.

The causal state-and-transcript count is elementary; the contribution here is to register cut, initial state, service window, and accounting rule as a typed certificate and to compose overlaps by fractional packing. No genealogy may be inferred from static κ alone, and the Hong–Kung and area–time projections remain model scoped [9, 10].

4.5 Monotone reconstruction and linear duality

Typed units and normalization. CRG signatures are typed rather than dimensionless: state bytes, traffic bytes per operation, event counts, causal depth, work, energy, area, temperature, and reliability coordinates retain their declared units. Coordinates are not assumed commensurate. Convex combinations, norms, or scalar prices are interpreted only after the resource contract or technology ledger supplies the required normalization and unit conversion.

Definition 4.13 (Monotone interface-cost response). *For a continuous coordinatewise nondecreasing $\Phi : \mathbb{R}_+^{d(\mathcal{T})} \rightarrow \mathbb{R}_+$, define*

$$V_{F,\rho,\mathcal{T}}(\Phi) = \inf_{r \in \mathcal{R}_F^\rho(\mathcal{T})} \Phi(r),$$

with value $+\infty$ if the hierarchy is infeasible.

Theorem 4.14 (Reconstruction by monotone interface observations). *For every feasible hierarchy $\mathcal{T}$:*
 (a) *every continuous coordinatewise nondecreasing cost satisfies*

$$V_{F,\rho,\mathcal{T}}(\Phi) = \inf_{x \in \Gamma_F^\rho(\mathcal{T})} \Phi(x);$$

(b) *the full response $\Phi \mapsto V_{F,\rho,\mathcal{T}}(\Phi)$ reconstructs $\Gamma_F^\rho(\mathcal{T})$ exactly. In particular, for $a \in \mathbb{R}_+^{d(\mathcal{T})}$, let*

$$\Phi_a(x) = \|(x - a)_+\|_\infty.$$

Then

$$a \in \Gamma_F^\rho(\mathcal{T}) \iff V_{F,\rho,\mathcal{T}}(\Phi_a) = 0;$$

(c) *the decomposition-indexed response functional $(\mathcal{T}, \Phi) \mapsto V_{F,\rho,\mathcal{T}}(\Phi)$ and the family of upper obligation regions $\{\Gamma_F^\rho(\mathcal{T})\}_\mathcal{T}$ determine one another, up to coordinate relabelings declared by ρ .*

Conversely, those regions determine every query in this observation class. The completeness claim does not extend automatically to objectives depending on a realization fiber, causal timing, switching activity, or physical embedding.

Proof. Monotonicity gives $\inf_{\text{up}} \mathcal{R} \Phi = \inf_{\mathcal{R}} \Phi$: increasing any provisioned coordinate cannot improve the cost. Continuity preserves the infimum under closure, proving (a).

If $a \in \Gamma$, then $\Phi_a(a) = 0$. Conversely, if the infimum is zero, choose $x_n \in \Gamma$ with $\|(x_n - a)_+\|_\infty \rightarrow 0$ and set $y_n = x_n \vee a$ coordinatewise. Upper closure gives $y_n \in \Gamma$, while $y_n \rightarrow a$; closedness gives $a \in \Gamma$. Applying this membership test on every admissible hierarchy proves (c), and the converse follows from (a). $\square$

Definition 4.15 (Lower linear support cost). *For $w \in \mathbb{R}_+^{d(\mathcal{T})}$, define*

$$h_{F,\rho,\mathcal{T}}(w) = \inf_{c \in \mathcal{K}_F^\rho(\mathcal{T})} \langle w, c \rangle.$$

Theorem 4.16 (Linear-cost-body duality). *Let $\mathcal{K} \subseteq \mathbb{R}_+^m$ be nonempty, closed, convex, and upper closed, and put $h_{\mathcal{K}}(w) = \inf_{x \in \mathcal{K}} \langle w, x \rangle$ for $w \geq 0$. Then*

$$\mathcal{K} = \{x \in \mathbb{R}_+^m : \langle w, x \rangle \geq h_{\mathcal{K}}(w) \text{ for every } w \in \mathbb{R}_+^m\}.$$

Thus the optimum under every additive nonnegative interface-price system uniquely determines the linear-cost body [12].

Proof. Every $x \in \mathcal{K}$ satisfies the inequalities by definition. Conversely, let $x \notin \mathcal{K}$. Strong separation gives a and α with

$$\langle a, x \rangle < \alpha \leq \langle a, y \rangle \quad \text{for every } y \in \mathcal{K}.$$

The separator must satisfy $a \geq 0$: if $a_j < 0$, upper closure would put $y + te_j$ in $\mathcal{K}$ for every $t \geq 0$, driving $\langle a, y + te_j \rangle$ to $-\infty$. Hence a is an allowed price vector and $\langle a, x \rangle < h_{\mathcal{K}}(a)$. $\square$

Corollary 4.17 (Linear-price completeness). *For a fixed hierarchy, $\mathcal{K}_F^\rho(\mathcal{T})$ and the response $h(w)$ determine one another, and distinct bodies are separated by a nonnegative price vector. Convexification is exact for additive linear interface prices; general monotone costs require Γ , while objectives distinguishing realizations with one signature require $\mathcal{P}$.*

4.6 Safe commitment and insufficiency of scalar summaries

For a retained family $\emptyset \neq \mathcal{A} \subseteq \mathcal{P}_F^\rho(\mathcal{T})$ and an admissible signature objective C , define the commitment regret

$$\text{Regret}_C(\mathcal{A}) = \inf_{P \in \mathcal{A}} C(\sigma_{\mathcal{T}}(P)) - \inf_{P \in \mathcal{P}_F^\rho(\mathcal{T})} C(\sigma_{\mathcal{T}}(P)) \geq 0. \quad (4.1)$$

A reduction is safe for a declared decision class exactly when it leaves every optimum in that class unchanged. The next theorem identifies the corresponding geometric invariants.

Theorem 4.18 (Safe-Commitment Law: universal lossless retention). *Let $\emptyset \neq \mathcal{A} \subseteq \mathcal{P}_F^\rho(\mathcal{T})$ be a retained realization family and define*

$$\Gamma_{\mathcal{A}} = \text{cl up } \sigma_{\mathcal{T}}(\mathcal{A}), \quad \mathcal{K}_{\mathcal{A}} = \text{cl conv } \Gamma_{\mathcal{A}}.$$

Then:

- (i) $\mathcal{A}$ preserves the optimum of every continuous coordinatewise nondecreasing signature objective if and only if $\Gamma_{\mathcal{A}} = \Gamma_F^{\rho}(\mathcal{T})$;
- (ii) it preserves the optimum under every additive nonnegative price if and only if $\mathcal{K}_{\mathcal{A}} = \mathcal{K}_F^{\rho}(\mathcal{T})$; and
- (iii) if either equality fails, a member of the corresponding observation class has strictly positive commitment regret in the sense of Equation (4.1).

Proof. Region equality is sufficient. Conversely, equality of every monotone response includes the probes Φ_a from Theorem 4.14, so the two upper regions have identical membership. Equality of every nonnegative linear response reconstructs the same convex upper body by Theorem 4.16. If upper regions differ, choose a missing a and use Φ_a ; if convex bodies differ, nonnegative strong separation supplies a price with a strict gap. $\square$

Corollary 4.19 (Strict engineering-cost separation). *Let $\Gamma_{\mathcal{A}} \subsetneq \Gamma$ be closed upper sets generated by a retained family $\mathcal{A}$ and by the full legal realization family, respectively. Then there exists a continuous, coordinatewise strictly increasing, coercive cost $\Phi : \mathbb{R}_+^d \rightarrow \mathbb{R}_+$ such that*

$$\inf_{x \in \Gamma} \Phi(x) < \inf_{x \in \Gamma_{\mathcal{A}}} \Phi(x).$$

Consequently, loss of the monotone-decision geometry is witnessed by an engineering-style cost that penalizes every declared resource, not only by a membership probe.

Proof. Choose $a \in \Gamma \setminus \Gamma_{\mathcal{A}}$ and set

$$\delta := \inf_{x \in \Gamma_{\mathcal{A}}} \|(x - a)_+\|_{\infty}.$$

Closed upperness implies $\delta > 0$: otherwise a sequence in $\Gamma_{\mathcal{A}}$ would approach a after taking coordinatewise maxima with a , and closedness would place a in $\Gamma_{\mathcal{A}}$. Choose

$$0 < \varepsilon < \frac{\delta}{1 + \sum_i a_i}$$

and define

$$\Phi(x) := \|(x - a)_+\|_{\infty} + \varepsilon \sum_i x_i.$$

The positive linear term makes Φ coordinatewise strictly increasing and coercive. Every $x \in \Gamma_{\mathcal{A}}$ satisfies $\Phi(x) \geq \delta$, whereas $\Phi(a) = \varepsilon \sum_i a_i < \delta$. Since $a \in \Gamma$, strict separation follows. $\square$

Safe-commitment principle. Within the declared observation class, a realization may be discarded only when the relevant geometry is unchanged. Equivalently, discard a realization only when every future decision in that class is unable to detect that it is gone.

For a finite registry, universal monotone retention means that every discarded signature is dominated by a retained one. Linear-price retention permits the weaker test

$$\mathcal{R}_F^{\rho}(\mathcal{T}) \subseteq \text{conv}(\sigma_{\mathcal{T}}(\mathcal{A})) + \mathbb{R}_+^{d(\mathcal{T})}.$$

These statements are universal only relative to their named observation classes; a fixed finite technology ledger can require less data.

Theorem 4.20 (Strict insufficiency of marginal and scalar summaries). *There are two closed upper feasible regions with identical coordinate floors, minimum total traffic, and minimum bottleneck width, but different technology-price responses. Let*

$$\Gamma_A = \text{up}\{(1, 2), (2, 1)\}, \quad \Gamma_B = \text{up}\{(1, 3), (2, 1)\}.$$

Both satisfy

$$(\inf x_1, \inf x_2) = (1, 1), \quad \inf(x_1 + x_2) = 3, \quad \inf \max\{x_1, x_2\} = 2.$$

Yet for $w = (10, 1)$,

$$\inf_{x \in \Gamma_A} \langle w, x \rangle = 12, \quad \inf_{x \in \Gamma_B} \langle w, x \rangle = 13.$$

The jointly realizable region therefore retains operational information lost by coordinatewise minima, total width, and bottleneck width.

Proof. Evaluate the two Pareto-minimal points of each upper region. Under $w = (10, 1)$, $(1, 2)$ minimizes Γ_A at 12, while $(1, 3)$ minimizes Γ_B at 13; $(2, 1)$ costs 21 in both. $\square$

Example 4.21 (Why the exact region cannot be convexified universally). *Suppose the only exact signatures are $r = (0, 2)$ and $s = (2, 0)$. Their convex linear-cost body contains $(1, 1)$. Every linear price has the same infimum on the two raw points and on the body, but the monotone convex objective $\Phi(c) = \max\{c_1, c_2\}$ has exact optimum 2 and convex-relaxation optimum 1. Thus $\mathcal{K}$ is exact for linear prices, $\mathcal{R}$ preserves discrete joint alternatives, and $\mathcal{P}$ is required to preserve architecture identity within each signature fiber.*

Proposition 4.22 (Price reversal for incomparable signatures). *If $r, s \in \mathbb{R}_+^m$ are incomparable in coordinate order, there are nonnegative prices w, w' such that*

$$\langle w, r \rangle < \langle w, s \rangle, \quad \langle w', s \rangle < \langle w', r \rangle.$$

No price-independent monotone scalar ranking can therefore agree with every nonnegative linear price system.

Proof. Incomparability gives coordinates i, j with $r_i < s_i$ and $r_j > s_j$. Take $w = e_i$ and $w' = e_j$. $\square$

5 Stable dilation limits and finite hierarchy phase

Definition 5.1 (Regular variation). *A measurable positive function R is regularly varying with index p , written $R \in \text{RV}_p$, if*

$$\lim_{g \rightarrow \infty} \frac{R(\lambda g)}{R(g)} = \lambda^p \quad \text{for every } \lambda > 0.$$

A function in RV_0 is slowly varying.

Theorem 5.2 (Stable-scale limit). *Let $R : (g_0, \infty) \rightarrow (0, \infty)$ be measurable. Suppose that for every $\lambda > 0$ the finite positive limit*

$$Q(\lambda) = \lim_{g \rightarrow \infty} \frac{R(\lambda g)}{R(g)}$$

exists. Then there is a unique $p \in \mathbb{R}$ such that

$$Q(\lambda) = \lambda^p$$

for all $\lambda > 0$. Consequently,

$$\boxed{R(g) = g^p \ell(g)}$$

for a slowly varying function ℓ .

Proof. For $\lambda, \mu > 0$,

$$\frac{R(\lambda \mu g)}{R(g)} = \frac{R(\lambda \mu g)}{R(\mu g)} \frac{R(\mu g)}{R(g)}.$$

Taking limits gives $Q(\lambda \mu) = Q(\lambda)Q(\mu)$. The pointwise limit of measurable functions is measurable, so Q is a positive measurable multiplicative function. Setting $\varphi(t) = \log Q(e^t)$ gives a measurable additive function and hence $\varphi(t) = pt$ for a unique p . Therefore $Q(\lambda) = \lambda^p$. Defining $\ell(g) = R(g)/g^p$ gives $\ell(\lambda g)/\ell(g) \rightarrow 1$ [1]. $\square$

Corollary 5.3 (Pure power as an exact fixed point). *If $R(\lambda g) = \lambda^p R(g)$ for every $g, \lambda > 0$, then*

$$R(g) = t g^p$$

for $t = R(1)$. Conversely, every pure power has exact scale covariance.

The exact power is therefore one fixed point. Stable dilation limits also admit a slowly varying envelope, which can remain numerically important over every realizable design scale.

5.1 Preferred dilation factors

Digital hierarchies often repeat only at a declared factor $b > 1$. Covariance under one factor is weaker than regular variation.

Theorem 5.4 (Discrete scale covariance). *Fix $b > 1$. A positive function satisfies*

$$R(bg) = b^p R(g) \quad \text{for all } g > 0$$

if and only if

$$R(g) = g^p \psi(\log_b g)$$

for a positive 1-periodic function ψ .

Proof. Define $\psi(t) = b^{-pt} R(b^t)$. The covariance relation gives $\psi(t+1) = \psi(t)$. Substitution proves the converse. $\square$

This is the discrete-scale-invariance mechanism behind log-periodic corrections [2]. A binary or quadtree hierarchy can have a stable mean exponent while retaining a phase oscillation in logarithmic coordinates.

Theorem 5.5 (Two-scale criterion). *Let R be eventually nondecreasing. Suppose there exist $a, b > 1$ and $p \geq 0$ such that $\log a / \log b$ is irrational and*

$$\frac{R(ag)}{R(g)} \rightarrow a^p, \quad \frac{R(bg)}{R(g)} \rightarrow b^p.$$

Then $R \in \text{RV}_p$.

Proof. The subgroup $\{a^m b^n : m, n \in \mathbb{Z}\}$ is dense in $\mathbb{R}_+$. Repeated application of the two limits, including reciprocal limits for negative powers, gives $R(hg)/R(g) \rightarrow h^p$ for every subgroup element h . For arbitrary $\lambda > 0$, choose subgroup elements $h_- < \lambda < h_+$ arbitrarily close to λ . Eventual monotonicity gives

$$\frac{R(h_-g)}{R(g)} \leq \frac{R(\lambda g)}{R(g)} \leq \frac{R(h_+g)}{R(g)}.$$

First take $g \rightarrow \infty$ and then $h_{\pm} \rightarrow \lambda$. $\square$

One preferred factor can therefore preserve hierarchy phase; stable scaling at two incommensurate factors removes that ambiguity.

5.2 Normalized dilation action and phase ratios

Definition 5.6 (Normalized dilation action). *For a positive measurable profile, considered modulo a positive amplitude, choose the representative $R(1) = 1$ and define*

$$(\mathcal{D}_a R)(x) = \frac{R(ax)}{R(a)}, \quad a > 0.$$

Then $\mathcal{D}_a \mathcal{D}_c = \mathcal{D}_{ac}$.

Proposition 5.7 (Fixed points and scale cycles). *A positive measurable profile is fixed by every $\mathcal{D}_a$ exactly when $R(x) = x^p$. For a declared hierarchy factor $b > 1$, it is fixed by $\mathcal{D}_b^h$ exactly when*

$$R(x) = x^p \psi(\log_b x)$$

for a positive h -periodic ψ normalized by $\psi(0) = 1$. Finally, $\mathcal{D}_a R \rightarrow x^p$ locally uniformly as $a \rightarrow \infty$ exactly when $R \in \text{RV}_p$.

Proof. Full invariance is the measurable multiplicative equation $R(ax) = R(a)R(x)$, whose positive solutions are powers. Discrete invariance follows by setting $\psi(t) = b^{-pt} R(b^t)$ and imposing h -periodicity; the converse is immediate. The last statement is the uniform convergence theorem for regular variation [1]. $\square$

The following result characterizes the observable scalar sequence. It does not infer the latent mechanism that generated that sequence.

Theorem 5.8 (Phase-ratio characterization). *Fix $g_k = g_0 b^k$ and a positive sequence Y_k with its canonical log-linear interpolation. For an integer $h \geq 1$, the phasewise limits*

$$\frac{Y_{nh+j+1}}{Y_{nh+j}} \longrightarrow c_j > 0, \quad j = 0, \dots, h-1,$$

exist if and only if

$$\boxed{Y(g) = Y(g_0)(g/g_0)^p L(g) \psi(\log_b(g/g_0))},$$

where

$$p = \frac{\log \prod_{j=0}^{h-1} c_j}{h \log b},$$

$L \in \text{RV}_0$, and ψ is positive, log-linearly interpolated, and h -periodic. Under this interpolation, $Y \in \text{RV}_p$ exactly when all $c_j = b^p$. If

$$\sum_k \left| \log \frac{Y_{k+1}/Y_k}{c_{k \bmod h}} \right| < \infty,$$

then $L(g) \rightarrow L_\infty \in (0, \infty)$.

Proof. Set $a_0 = 1$ and $a_{j+1} = a_j c_j b^{-p}$; the definition of p gives $a_h = a_0$. For $k = nh + j$, define

$$L_k = \frac{Y_k}{Y_0 b^{pk} a_j}.$$

Then $L_{k+1}/L_k \rightarrow 1$. An interval from g to λg crosses only a bounded number of hierarchy cells, and the logarithmic increment on each late cell tends to zero. Hence the log-linear interpolation of L_k is slowly varying. Periodically interpolate a_j to obtain ψ . Substitution proves the reverse implication. A nonconstant ψ leaves distinct phase-subsequence ratios, whereas absolute summability makes $\log L_k$ a convergent series. $\square$

The adjacent log slopes

$$s_k = \frac{\log(Y_{k+1}/Y_k)}{\log b}$$

therefore converge to one value in the regularly varying class, converge to an h -cycle in the finite-phase class, and do neither outside those output classes. Phase-ratio convergence is observable; finite-dimensional positive mixing, introduced next, is one sufficient explanation rather than a necessary one.

6 Recursive mechanisms and certified scaling basins

The output characterizations in [Section 5](#) do not say why a legal realization should generate stable ratios. Recursive composition of typed interface modes supplies one sufficient mechanism.

Theorem 6.1 (Summable scalar distortion). *Let $b > 1$, $c > 0$, and suppose a positive profile on $g_k = g_0 b^k$ obeys*

$$R(g_{k+1}) = cR(g_k)(1 + \varepsilon_k), \quad \varepsilon_k > -1,$$

with $\sum_{k \geq 0} |\varepsilon_k| < \infty$. Then $\prod_{j < k} (1 + \varepsilon_j)$ converges to a finite positive limit and

$$R(g_k) = \Theta(c^k) = \Theta(g_k^p), \quad p = \log_b c.$$

Under the canonical log-linear interpolation, the profile is regularly varying with index p and in fact has an asymptotically constant reduced amplitude.

Proof. Iteration gives

$$R(g_k) = R(g_0) c^k \prod_{j < k} (1 + \varepsilon_j).$$

Absolute summability, positivity of every factor, and the convergence of $\sum_j \log(1 + \varepsilon_j)$ after a finite prefix imply a positive finite product limit. Since $c^k = (g_k/g_0)^{\log_b c}$, the result follows. $\square$

The vector mechanism retains mode composition and hierarchy phase rather than reducing the recursion to one scalar multiplier.

Definition 6.2 (Positive interface-mode cocycle). *At scales $g_k = g_0 b^k$, let $v_k \in \mathbb{R}_{++}^m$ collect typed interface modes, let $u \in \mathbb{R}_{++}^m$ be the declared observation vector, and write*

$$v_{k+1} = A_k v_k + \eta_k, \quad Y_k = u^\top v_k, \quad z_k = \frac{v_k}{u^\top v_k}.$$

The relative recursion error is

$$\delta_k = \max_i \frac{|(\eta_k)_i|}{(A_k v_k)_i}.$$

Every sufficiently late A_k is nonnegative and maps the cone interior into itself, and each perturbed update remains in the cone interior. The law is asymptotically h -periodic when

$$A_{nh+j} \longrightarrow A^{(j)}, \quad j = 0, \dots, h-1,$$

in any fixed matrix norm. Each limiting phase map preserves the cone interior. With

$$M = A^{(h-1)} \dots A^{(0)},$$

the limiting law is projectively mixing after finite coarse graining when $M^\nu > 0$ entrywise for some fixed integer $\nu \geq 1$.

For $x, y > 0$, Hilbert's projective metric is

$$d_H(x, y) = \log \frac{\max_i x_i / y_i}{\min_i x_i / y_i}.$$

A positive matrix has finite projective diameter and contracts this metric [13, 14].

Theorem 6.3 (Perron–Floquet scaling on the certified mixing class). *Under Theorem 6.2, suppose $v_0 > 0$, $M^\nu > 0$ for some fixed ν , and $\delta_k \rightarrow 0$. Then:*

(i) *there is a unique positive u -normalized phase cycle $r_0, \dots, r_{h-1}$ such that*

$$z_{nh+j} \rightarrow r_j, \quad r_{j+1} = \frac{A^{(j)} r_j}{u^\top A^{(j)} r_j},$$

for every admitted positive initial trajectory, with phase indices read modulo h ;

(ii) *with $c_j = u^\top A^{(j)} r_j$,*

$$\frac{Y_{nh+j+1}}{Y_{nh+j}} \rightarrow c_j, \quad \prod_{j=0}^{h-1} c_j = \text{spr}(M);$$

(iii) *the profile has*

$$Y(g) = Y(g_0)(g/g_0)^p L(g) \psi(\log_b(g/g_0)), \quad p = \frac{\log \text{spr}(M)}{h \log b},$$

where $L \in \text{RV}_0$ and ψ is positive and h -periodic; and

(iv) *at hierarchy phase j ,*

$$\mathcal{D}_{g_0 b^{nh+j}} Y(x) \longrightarrow x^p \frac{\psi(j + \log_b x)}{\psi(j)}$$

locally uniformly for $x > 0$.

If all $c_j = b^p$, then $Y \in \text{RV}_p$. A nonconstant minimal phase sequence yields asymptotic discrete scale invariance. If

$$\sum_k \left(\|A_k - A^{(k \bmod h)}\| + \delta_k \right) < \infty,$$

then $L(g) \rightarrow L_\infty \in (0, \infty)$.

Proof. Write $[x]_u = x/(u^\top x)$. On the normalized cone section $\{z > 0 : u^\top z = 1\}$, define the unperturbed maps

$$\mathcal{F}_k(z) = \frac{A_k z}{u^\top A_k z}.$$

Because $\delta_k \rightarrow 0$, eventually $\delta_k < 1$. The componentwise relative error bound gives

$$(1 - \delta_k)A_k v_k \leq A_k v_k + \eta_k \leq (1 + \delta_k)A_k v_k.$$

Consequently,

$$d_H(z_{k+1}, \mathcal{F}_k(z_k)) \leq \log \frac{1 + \delta_k}{1 - \delta_k} = O(\delta_k). \quad (6.1)$$

This is the required perturbation bookkeeping. It permits signed η_k provided the relative bound and cone-interior condition hold.

Group the recursion into periods and then into primitive blocks:

$$B_n = A_{nh+h-1} \cdots A_{nh}, \quad C_n = B_{n\nu+\nu-1} \cdots B_{n\nu}.$$

Then $B_n \rightarrow M$ and $C_n \rightarrow M^\nu > 0$ entrywise. After a finite prefix, the νh -step phase-zero maps have uniformly bounded projective diameter and one Birkhoff contraction coefficient $\kappa_{\text{ctr}} < 1$. Let r_0 be the u -normalized Perron direction of M , equivalently of M^ν , and set $e_n = d_H(z_{n\nu h}, r_0)$. Nonnegative cone-interior maps are nonexpansive in Hilbert metric. Applying (6.1) through one finite block, using $C_n \rightarrow M^\nu$ and $[M^\nu r_0]_u = r_0$, gives

$$e_{n+1} \leq \kappa_{\text{ctr}} e_n + \epsilon_n, \quad \epsilon_n \rightarrow 0.$$

Hence $e_n \rightarrow 0$. Propagation through the intervening period and phase maps gives

$$z_{nh+j} \rightarrow r_j, \quad r_{j+1} = [A^{(j)} r_j]_u.$$

Because $[M r_0]_u = r_0$, the cycle closes. Primitivity of M makes r_0 , and therefore the whole phase cycle, unique and independent of v_0 .

The scalar gain satisfies

$$\frac{Y_{k+1}}{Y_k} = u^\top A_k z_k + \frac{u^\top \eta_k}{Y_k}. \quad (6.2)$$

The perturbation term is controlled without assuming cancellation:

$$\left| \frac{u^\top \eta_k}{Y_k} \right| \leq \delta_k u^\top A_k z_k = o(1). \quad (6.3)$$

Matrix and state convergence therefore yield the phasewise gains $c_j > 0$. The cycle relation is $A^{(j)} r_j = c_j r_{j+1}$; multiplying over one period gives

$$M r_0 = \left(\prod_j c_j \right) r_0.$$

Perron–Frobenius theory implies $\prod_j c_j = \text{spr}(M)$. The profile representation and dilation limits now follow from Theorems 5.7 and 5.8.

For the strengthened conclusion, discard a finite prefix. The normalized trajectory and limiting cycle then lie in a compact subset of the cone interior, where normalization and the positive gain functional are uniformly Lipschitz. If β_n is the sum of matrix and relative errors in the n th νh -block, summability sharpens the recurrence to

$$e_{n+1} \leq \kappa_{\text{ctr}} e_n + C \beta_n, \quad \sum_n \beta_n < \infty.$$

The contracted recurrence gives $\sum_n e_n < \infty$; finite-step propagation then gives

$$\sum_k d_H(z_k, r_{k \bmod h}) < \infty.$$

Equations (6.2)–(6.3) and Lipschitz continuity imply

$$\sum_k \left| \log \frac{Y_{k+1}/Y_k}{c_{k \bmod h}} \right| < \infty.$$

The last clause of Theorem 5.8 makes the slowly varying envelope converge to a positive finite constant. $\square$

Corollary 6.4 (Stationary primitive recursion). *If $A_k \rightarrow A$, A is primitive, and the relative recursion error tends to zero, then*

$$\frac{v_k}{u^\top v_k} \rightarrow r, \quad \frac{Y_{k+1}}{Y_k} \rightarrow \text{spr}(A), \quad Y \in \text{RV}_{\log_b \text{spr}(A)}.$$

If the matrix and relative errors are summable, then $Y(g) \sim Cg^{\log_b \text{spr}(A)}$.

Proof. Apply [Theorem 6.3](#) with $h = 1$. □

Remark 6.5 (Scope of the basin certificate). *The theorem classifies every admitted trajectory in the asymptotically finite-periodic, projectively mixing, finite-mode class with vanishing relative error. It is an in-basin result after membership hypotheses are supplied. It does not say that positivity, finite mode dimension, projective mixing, or periodicity is necessary for an observed regularly varying profile. The scalar equivalence in [Theorem 5.8](#) characterizes output ratios, not latent dynamics. Nonlinear, infinite-mode, stochastic, imprimitive, quasiperiodic, or nonpositive mechanisms remain outside this certificate and may require different theorems.*

Definition 6.6 (Declared cap response). *A cap response is a continuous nondecreasing function $\Phi : (0, \infty) \rightarrow (0, 1]$ satisfying*

$$\Phi(z)/z \rightarrow 1 \quad (z \downarrow 0), \quad \Phi(z) \rightarrow 1 \quad (z \rightarrow \infty).$$

For an interior profile I and declared terminal budget B , define

$$\mathcal{C}_B[I](g) = B\Phi(I(g)/B).$$

The hard cap is $\Phi(z) = \min\{z, 1\}$; a soft rational cap is $\Phi(z) = z/(1+z)$.

Theorem 6.7 (Boundary crossover). *Let $p > 0$, let $B_n \rightarrow \infty$, and choose $g_n \rightarrow \infty$ with $I(g_n)/B_n \rightarrow 1$.*

(i) If $I \in \text{RV}_p$, then

$$\boxed{\frac{\mathcal{C}_{B_n}[I](g_n x)}{B_n} \longrightarrow \Phi(x^p)}$$

locally uniformly for $x > 0$. The hard-cap limit is $\min\{x^p, 1\}$.

(ii) If instead

$$I(g) = g^p L(g) \psi(\log_b g),$$

where $L \in \text{RV}_0$ and ψ is positive, continuous, and h -periodic, then every subsequence satisfying $\log_b g_n \bmod h \rightarrow s$ has the phase-indexed limit

$$\frac{\mathcal{C}_{B_n}[I](g_n x)}{B_n} \longrightarrow \Phi\left(x^p \frac{\psi(s + \log_b x)}{\psi(s)}\right).$$

Proof. Uniform convergence for regular variation gives $I(g_n x)/I(g_n) \rightarrow x^p$ locally uniformly. Combine it with $I(g_n)/B_n \rightarrow 1$ and continuity of Φ . In the finite-phase branch, slow variation cancels while the periodic factor retains the limiting hierarchy phase. □

Corollary 6.8 (Crossover slope). *In the regularly varying branch of [Theorem 6.7](#), additionally suppose*

$$\frac{gI'(g)}{I(g)} \rightarrow p$$

and $\Phi \in C^1$ near x^p . Then the local logarithmic slope at $g_n x$ converges to

$$\boxed{pE_\Phi(x^p), \quad E_\Phi(z) = \frac{z\Phi'(z)}{\Phi(z)}}.$$

For the hard cap, the statement holds away from $x = 1$, with one-sided slopes at the kink.

Theorem 6.9 (Partial converse from phase-rich cap collapse). *Fix $p > 0$ and let $B_n > 0$. Let I be positive and eventually nondecreasing, let $g_n \uparrow \infty$ satisfy $g_{n+1}/g_n \rightarrow 1$, and suppose $I(g_n)/B_n \rightarrow 1$. Let J be an open multiplier interval containing 1. Assume Φ restricts to a homeomorphism on an interval containing $\{x^p : x \in J\}$ and all sufficiently late normalized arguments. If*

$$\frac{\mathcal{C}_{B_n}[I](g_n x)}{B_n} \longrightarrow \Phi(x^p)$$

locally uniformly for $x \in J$, then $I \in \text{RV}_p$.

Proof. Continuity of Φ^{-1} gives

$$\frac{I(g_n x)}{B_n} \rightarrow x^p, \quad \frac{I(g_n x)}{I(g_n)} \rightarrow x^p$$

locally uniformly on J . For arbitrary $t \rightarrow \infty$, bracket t between adjacent g_n, g_{n+1} . Eventual monotonicity, the dense-anchor condition, and slightly smaller and larger multipliers in J squeeze $I(tx)/I(t)$ to x^p for $x \in J$. Finite products and reciprocals of an open interval containing 1 cover $(0, \infty)$, so the dilation limit holds for every positive multiplier. $\square$

The cap shape is a declared model component represented by Φ . A purely dyadic grid samples a period-one function of $\log_2 g$ at one phase and cannot meet the dense-anchor premise of the converse. The two-sided converse also excludes the hard cap because it is noninjective after saturation; a one-sided version can use an unsaturated interval. Terminal budgets and pinned boundary conditions are coordinates of the resource contract ρ , not measurement noise.

Definition 6.10 (Finite semialgebraic architecture registry). *Let $\Theta \subseteq \mathbb{R}^p$ be a semialgebraic technology domain. A finite registry consists of architecture classes $a \in \mathcal{A}$ and finite witness sets W_a . A witness $\omega \in W_a$ records every discrete realization, allocation, layout, and schedule choice. Its feasibility is described by finitely many polynomial or rational inequalities whose denominators have fixed sign on Θ ; its cost is assembled from rational functions by finitely many min and max operations. Infeasible witnesses have cost $+\infty$. Put*

$$C_a(\theta) = \min_{\omega \in W_a} C_{a\omega}(\theta), \quad A(\theta) = \left\{ a : C_a(\theta) = \min_b C_b(\theta) < +\infty \right\}.$$

If every registered class is infeasible, $A(\theta) = \emptyset$.

Theorem 6.11 (Finite technology-phase stratification). *Every registry satisfying Theorem 6.10 admits a finite partition of Θ into connected, relatively open semialgebraic sign cells, with relative openness taken in each cell's supporting stratum. On every cell the following complete set-valued labels are constant:*

- (i) feasible classes and witnesses;
- (ii) every active rational branch and bottleneck set, including ties;
- (iii) the optimal witness set within every class; and
- (iv) the globally winning architecture set $A(\theta)$.

Every label change lies on a feasibility wall, branch or bottleneck exchange, witness exchange, architecture tie, or boundary of Θ .

Proof. Clear denominators on regions where their signs are fixed. Collect the finite family of feasibility polynomials, branch-comparison polynomials, within-class and cross-class cost differences, and defining polynomials of Θ . A sign-invariant semialgebraic cell decomposition exists for this finite family [11]; refine each stratum into connected cells. Every comparison then has one constant sign in $-, 0, +$. Retaining the zero sign preserves complete tied branch, bottleneck, witness, and architecture sets instead of inserting a hidden tie-break. The listed labels are consequently constant, and they can change only when a defining sign or domain stratum changes. $\square$

Corollary 6.12 (Certified full-label phase radius). *Let $\mathcal{P}$ contain every cleared feasibility polynomial, branch difference, within-class witness difference, and cross-class difference used in Theorem 6.11. Suppose $\theta_0 \in \Theta$, every $p \in \mathcal{P}$ is nonzero at θ_0 , and p is L_p -Lipschitz on $\bar{B}(\theta_0, r_0) \cap \Theta$. Then the complete phase label is unchanged throughout $B(\theta_0, r) \cap \Theta$ whenever*

$$r < \min \left\{ r_0, \min_{p \in \mathcal{P}: L_p > 0} \frac{|p(\theta_0)|}{L_p} \right\},$$

where the inner minimum is $+\infty$ if every $L_p = 0$.

Proof. For $|\theta - \theta_0|_2 < |p(\theta_0)|/L_p$, the Lipschitz inequality prevents p from reaching zero, so every defining sign is preserved. Intersecting with Θ also covers a point on the admissible-domain boundary. The radius controls losing-class witness exchanges and presently infeasible witnesses as well as the global winner. $\square$

A one-axis distance to a tie wall is therefore only a directional diagnostic; it is not the full-label radius unless every defining sign and the geometry of Θ have been included.

6.1 Certified inner and outer descriptions

Definition 6.13 (Certified outer relaxation). *For each hierarchy let*

$$\text{Adm}_\tau(\mathcal{T}) \subseteq \{(P, \phi) : P \in \mathfrak{R}_F^\rho(\mathcal{T}), \phi \in \text{Emb}_\tau(P, \mathcal{T})\}$$

be the exact admissible pairs. A lower-bounding relaxation consists of a superset

$$\text{Adm}_\tau^{\text{out}}(\mathcal{T}) \supseteq \{(\sigma_\tau(P), \phi) : (P, \phi) \in \text{Adm}_\tau(\mathcal{T})\}$$

and an optimistic objective satisfying

$$\Phi_{\tau, \mathcal{T}}^{\text{out}}(\sigma_\tau(P), \phi) \leq \Phi_\tau(P, \phi)$$

for every exact admissible pair.

Theorem 6.14 (Certified sandwich and safe pruning). *Let $\mathcal{J}_\in$ be a finite set of actual legal realization-embedding pairs. Then*

$$\boxed{\text{LB}_\tau \leq \text{OPT}_\tau^\rho(F) \leq \text{UB}_\tau,}$$

where

$$\text{LB}_\tau = \inf_{\mathcal{T}} \inf_{(z, \phi) \in \text{Adm}_\tau^{\text{out}}(\mathcal{T})} \Phi_{\tau, \mathcal{T}}^{\text{out}}(z, \phi), \quad \text{UB}_\tau = \min_{(P, \phi) \in \mathcal{J}_{\text{in}}} \Phi_\tau(P, \phi).$$

A candidate class may be discarded whenever its scope-matched certified lower bound is strictly larger than the constructive incumbent.

Proof. Every exact admissible pair maps into the relaxed set with no larger relaxed cost, so the relaxed infimum is a lower bound. Every member of $\mathcal{J}_\in$ is an exact feasible design, so its best evaluated cost is an upper bound. A class whose lower bound exceeds that incumbent cannot contain an in-scope optimizer. $\square$

The scope check is part of the proof obligation. Computation, candidate class, objective and units, hierarchy, resource contract, technology contract, assumptions, and validity domain must match the decision being made. Any dependency change invalidates the affected certificate and reopens the corresponding branch. The retained object is the bracket and its dependency ledger, not merely the lowest evaluated point.

6.2 Lossless retention for a technology domain

Theorem 6.15 (Argmin-hitting retention). *Let $\mathcal{P}$ be a finite registered realization set and let Θ be any technology domain on which every technology has a finite attained optimum. Define*

$$\mathcal{M}_\theta = \underset{P \in \mathcal{P}}{\text{argmin}} C_\theta(P).$$

A retained family $\mathcal{A} \subseteq \mathcal{P}$ preserves the optimum at every technology in Θ if and only if

$$\boxed{\mathcal{A} \cap \mathcal{M}_\theta \neq \emptyset \quad \text{for every } \theta \in \Theta.}$$

A singleton is lossless exactly when $\bigcap_{\theta \in \Theta} \mathcal{M}_\theta \neq \emptyset$.

Proof. At one technology, the retained and full minima agree exactly when a retained realization attains the full minimum. Quantifying this equivalence over Θ proves both statements. $\square$

This is the exact statistic for a named technology domain. It is weaker than universal preservation for all continuous monotone objectives, which requires preserving the complete upper obligation region, and weaker than preservation for every additive nonnegative price, which requires preserving its convex upper body.

7 Closure architecture and dependency map

The closure theorems form a directed dependency graph rather than a list of independent claims.

- C1. Semantic closure.** Contextual quotients identify the exact state required by a declared side of a computation. Circuit–protocol translations place that state inside a general causal realization family.
- C2. Interactive closure.** The directional range lemma converts the receiver’s output range into a forward-traffic converse. For finite-field linear maps, cut ranks identify that range and local quotient composition constructs one realization attaining every edge simultaneously.
- C3. Carrier and temporal closure.** Carrier/network-code equivalence transfers the realization semantics to fixed directed networks. Cut-state conservation extends static traffic to retained state and service windows; fractional packing composes overlapping certificates.
- C4. Observation closure.** Monotone probes reconstruct Γ and linear prices reconstruct $\mathcal{K}$. These reconstruction results turn equality of decision responses into equality of geometric objects.
- C5. Commitment closure.** The Safe-Commitment Law applies the reconstruction theorems to a retained realization family and gives exact necessary-and-sufficient preservation conditions.
- C6. Finite technology closure.** Semialgebraic sign decomposition closes feasibility, active branch, bottleneck, witness, and winner labels on finite technology cells. Argmin hitting closes retention over a named technology ledger.
- C7. Scale closure.** Stable dilation limits and phase-ratio characterization close the observable output classes. Perron–Floquet mixing supplies a sufficient finite-mode mechanism, and phase-rich cap collapse gives a converse from normalized crossover to regular variation.

The architecture separates three uses of the word exact. A lower bound is exact when matched by a legal witness in the same contract. An observation quotient is exact when it reconstructs every decision in its declared class. A finite phase decision is exact when the witness registry, objective, and technology domain are completely enumerated. These are distinct closure statements joined by the same computation-first quantifier order.

8 Conclusion

The closure theorems establish that computation-realization geometry is both a constructive and a converse theory. In exact finite fragments it has a simultaneously attained directional lower corner, a carrier/network-code correspondence, a native state–traffic conservation law, observation-complete quotients, and a necessary-and-sufficient retention theorem.

The Safe-Commitment Law is the central closure statement. Continuous monotone resource decisions see exactly Γ ; additive nonnegative prices see exactly $\mathcal{K}$. Equality of the corresponding retained object is therefore both necessary and sufficient for universal preservation in the declared class, and separation constructs the decision that witnesses regret when preservation fails.

The phase and scaling results extend the same logic. Finite technology registries decompose into complete semialgebraic phase cells, and retained families are lossless exactly when they hit every attained argmin. Stable scale ratios generate regular-variation and finite-phase output classes; projective positive dynamics and cap response provide explicit mechanisms and converses. Achievability, obligation, observation, retention, technology phase, and scaling thus close on one computation-first realization object.

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