

A Generalized Rent Law from Computation-Realization Geometry

Scaling Fixed Points, Hierarchy Phase, and Finite-Scale Diagnostics

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Abstract

This paper develops a conditioned scaling theory for Computation–Realization Geometry. A Rent-like profile is defined as the multiscale observation of one legal realization under a declared hierarchy, accounting convention, estimator, and fit window. Stable dilation limits yield regular variation; preferred digital dilations permit periodic hierarchy phase; phase-ratio limits characterize power–slow-variation–phase decompositions; and positive periodic recursions with vanishing relative perturbations yield Perron–Floquet direction cycles. For heterogeneous modes, the local slope is a traffic-weighted mean and its curvature separates between-mode slope variance from within-mode drift. Interface reach, boundary caps, finite-window bias, concentration, and demand–service scale matching are derived within the same framework. Registered public-netlist diagnostics demonstrate hierarchy and estimator sensitivity, exactly close the declared same-hierarchy accounting identity as a software consistency check, and show nontrivial cross-hierarchy transfer failure. The result reclassifies a Rent exponent from a portable intrinsic scalar to a conditioned observation of a larger realization geometry.

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1 Introduction

Rent’s rule made the multiscale structure of circuit connectivity measurable. Its enduring lesson is that communication pressure has structure across scale. A measured Rent profile, however, is produced by a realized graph, a hierarchy, a terminal convention, an accounting rule, an estimator, and a scale window. Modern heterogeneous systems make those conditions part of the design itself: communication modes mix, hierarchy can retain phase, interface populations can overtake core traffic, and terminal limits can bend an otherwise stable interior law.

This paper develops a generalized scaling theory from *computation-realization geometry* (CRG). CRG begins with a declared computation, resource contract, legal realization, hierarchy, and typed observation. A scale profile is one projection of one jointly attainable signature. It is never assembled by independently minimizing different scales or by transferring an exponent across incompatible hierarchies.

The theory separates output class, mechanism, and estimator. Stable dilation limits yield regular variation. One preferred hierarchy factor permits a periodic function of log scale; two incommensurate factors recover full regular variation under monotonicity. Phasewise adjacent-ratio limits characterize the observable finite-phase class. Projectively mixing asymptotically periodic positive cocycles supply a generative Perron–Floquet mechanism with explicit perturbation control. Arithmetic composition gives exact local-slope and curvature identities; interface reach and terminal response give boundary laws; multiplicative-window estimation separates slowly varying bias from finite-design width.

A deterministic public-netlist study instantiates these distinctions. The same multiplier snapshot produces different fitted summaries under two registered hierarchies; measured same-hierarchy modes satisfy the exact arithmetic identity; global pure-mode fits and cross-hierarchy transfer fail by quantified amounts. A Transformer superblock then shows how near-quadratic attention work, linear feed-forward work, and an exponent-one package bridge produce one contract-sensitive scale and architecture phase.

The resulting theory replaces a single portable Rent exponent with a resource-conditioned family of scale profiles, mechanisms, estimators, and technology matches.

1.1 Principal results

- S1.** Stable dilation limits imply regular variation; exact fixed points are pure powers; one preferred hierarchy factor yields discrete scale invariance.
- S2.** A two-scale theorem and a phase-ratio characterization distinguish full regular variation from finite hierarchy phase.
- S3.** A Perron–Floquet theorem gives the unique phase cycle, exponent, and slowly varying envelope of a projectively mixing asymptotically periodic positive mode system.
- S4.** Arithmetic mode composition yields exact slope and curvature identities, including the decomposition into exponent variance and within-mode drift.
- S5.** Interface reach and declared cap response yield fanout-resolved boundary laws, normalized crossover limits, and a phase-rich converse.
- S6.** Windowed estimation and bounded heterogeneous composition separate finite-window bias from stochastic band width.
- S7.** The EPFL diagnostic protocol and Transformer profile demonstrate how the same formal calculus explains hierarchy sensitivity, finite-scale model failure, transfer error, and contract-dependent architecture transitions.

2 Historical and mathematical context

Rent’s rule began as an empirical relationship between pins and blocks in partitioned logic graphs [1]. Subsequent work connected Rent parameters to placement, wire length, intrinsic partition structure, heterogeneous subsystems, local variation, and estimator practice [2, 3, 4, 5, 6, 7]. This literature established that connectivity pressure has structure across scale and that hierarchy and measurement procedure matter.

The present theory places a Rent profile after the choice of computation realization, hierarchy, accounting rule, and observation. It therefore separates three questions that a scalar fit can conflate: the asymptotic output class of the measured profile, a dynamical mechanism capable of generating that class, and the finite-window estimator used to infer it. Regular variation supplies the output language for stable dilation ratios [8]; discrete scale invariance supplies hierarchy phase and log-periodic corrections [9]; Perron–Frobenius and Hilbert projective contraction supply a finite-mode mechanism for stable phase cycles [10, 11].

The mode-mixture identities are related to effective-exponent methods in heterogeneous scaling, but here they follow exactly from arithmetic composition of typed interface traffic. The curvature identity separates between-mode exponent variance from within-mode drift. Interface reach adds a fanout- and hierarchy-conditioned boundary mode, while cap response separates interior scaling from finite terminal saturation. These mechanisms unify phenomena historically treated as unrelated corrections to one Rent exponent.

Graph partitioning and separator theory provide hierarchies and cut measurements for realized graphs [12, 13, 14]. The public-netlist study uses deterministic RCM-based hierarchies as measurement instruments rather than optimal partitions. The resulting hierarchy sensitivity is therefore a controlled statement about the measurement map. CRG's contribution is to register that map as part of the scientific object and to connect its output to exact composition identities, regular-variation theorems, mechanism certificates, and falsifiable transfer tests.

Sequence and nonhomogeneous-product lineage. The sequence-level regular-variation results sit in the classical theory of regularly varying sequences [17, 18]. The positive-product mechanism is related to weak ergodicity and nonhomogeneous products of nonnegative matrices [19, 20, 21, 22].

3 CRG scale projections

Fix a terminalized computation F , a resource contract ρ , an admissible hierarchy $\mathcal{T}$, one legal realization P , and a typed signature map $\sigma_{\mathcal{T}}$. Computation-realization geometry retains the legal family $\mathcal{P}_F^{\rho}(\mathcal{T})$ and its exact jointly attainable signatures $\mathcal{R}_F^{\rho}(\mathcal{T}) = \sigma_{\mathcal{T}}(\mathcal{P}_F^{\rho}(\mathcal{T}))$. A scale profile is a registered projection of one signature $\sigma_{\mathcal{T}}(P)$ along nested boundaries. This precedence prevents a formally invalid construction in which a different realization is selected at each scale and the resulting points are reported as one architecture's law.

The word “geometry” therefore refers to the full feasible system from which a scale profile is observed. Scaling analysis can classify one projection; it does not replace realization identity, exact joint attainability, or the resource contract that made the projection legal.

Part I

Scaling laws from resource geometry

4 A scale profile from one legal realization

Fix a computation F , a resource contract ρ , an admissible recursive hierarchy $\mathcal{T}$, and one ρ -legal realization P . The foundations associate a typed interface signature with every boundary of $\mathcal{T}$ and collect the jointly attainable signatures in a realization region. A scale profile is a projection of one such jointly attainable signature; it is not a sequence obtained by minimizing each scale independently.

Let G denote total local work in the declared accounting unit. At hierarchy level k , let $\Pi_{\text{part},k}$ be the module partition and let $g_k = G/|\Pi_{\text{part},k}|$ be its nominal scale. The level is ζ -size-regular when every nonresidual module $M \in \Pi_{\text{part},k}$ satisfies

$$\zeta g_k \leq |M| \leq g_k, \quad 0 < \zeta \leq 1,$$

with the same ζ across the measured levels. Fix a typed projection π_χ of the interface signature—for example, fixed-bit terminal incidence per operation under a declared fanout convention—and write

$$T(M) = \pi_\chi([\sigma_{\mathcal{T}}(P)]_M) > 0$$

on the levels retained for logarithmic analysis.

Definition 4.1 (Fixed-realization level-mean profile). *For a size-regular level, define*

$$R_k = \frac{1}{|\Pi_{\text{part},k}|} \sum_{M \in \Pi_{\text{part},k}} T(M), \quad g_k = \frac{G}{|\Pi_{\text{part},k}|}.$$

The sequence $\{(g_k, R_k)\}$ is the level-mean scale profile of $(F, \rho, P, \mathcal{T}, \pi_\chi)$. Between adjacent hierarchy scales, its canonical interpolation is log-linear: $\log R$ is affine in $\log g$. Every derivative or local slope below refers to that interpolation unless a smooth continuum model is explicitly supplied.

For irregular partitions, either bin modules into fixed-ratio size classes or retain the full per-module scatter. The nominal value $G/|\Pi_{\text{part},k}|$ is not by itself a valid scale label when module sizes vary without control. Likewise, profiles with different terminal placement, hierarchy, traffic type, or accounting rule belong to different projections and must not be silently pooled.

The absolute terminal incidence at a level is

$$C(g_k) = \sum_{M \in \Pi_{\text{part},k}} T(M).$$

For $|\Pi_{\text{part},k}| \asymp G/g_k$,

$$R(g_k) = \frac{g_k}{G} C(g_k)$$

up to the fixed size-regularity constants. Circuit cut profiles, memory-traffic hierarchies, and component-plus-interface spectra are thus different typed projections of the same kind of fixed-realization object. The obligation region $\Gamma_F^\rho(\mathcal{T})$ may supply outer bounds on such profiles, but the profile itself is always traced through one legal realization across all measured levels. This coherence condition is fundamental: independently minimizing each scale would assemble a curve whose points may come from mutually incompatible algorithms, schedules, or hierarchies.

From incidence to a Rent-like profile. Suppose a level has m_k approximately equal modules, each of size $g_k \asymp G/m_k$, and aggregate external incidence $C(g_k)$. The mean incidence is

$$R(g_k) = \frac{C(g_k)}{m_k} \asymp \frac{g_k}{G} C(g_k).$$

If the aggregate incidence scales as $C(g) \asymp G g^{p-1} \ell(g)$, then the level mean scales as $R(g) \asymp g^p \ell(g)$. Thus the familiar pin-versus-block exponent is a projection of how aggregate boundary incidence changes along one hierarchy. The factor g/G is not cosmetic: it converts a level total into a per-module quantity and makes the measurement convention explicit.

5 Stable dilation limits and finite hierarchy phase

Definition 5.1 (Regular variation). *A measurable positive function R is regularly varying with index p , written $R \in \text{RV}_p$, if*

$$\lim_{g \rightarrow \infty} \frac{R(\lambda g)}{R(g)} = \lambda^p \quad \text{for every } \lambda > 0.$$

A function in RV_0 is slowly varying.

Theorem 5.2 (Stable-scale limit). *Let $R : (g_0, \infty) \rightarrow (0, \infty)$ be measurable. Suppose that for every $\lambda > 0$ the finite positive limit*

$$Q(\lambda) = \lim_{g \rightarrow \infty} \frac{R(\lambda g)}{R(g)}$$

exists. Then there is a unique $p \in \mathbb{R}$ such that

$$Q(\lambda) = \lambda^p$$

for all $\lambda > 0$. Consequently,

$$\boxed{R(g) = g^p \ell(g)}$$

for a slowly varying function ℓ .

Proof. For $\lambda, \mu > 0$,

$$\frac{R(\lambda \mu g)}{R(g)} = \frac{R(\lambda \mu g)}{R(\mu g)} \frac{R(\mu g)}{R(g)}.$$

Taking limits gives $Q(\lambda \mu) = Q(\lambda)Q(\mu)$. The pointwise limit of measurable functions is measurable, so Q is a positive measurable multiplicative function. Setting $\varphi(t) = \log Q(e^t)$ gives a measurable additive function and hence $\varphi(t) = pt$ for a unique p . Therefore $Q(\lambda) = \lambda^p$. Defining $\ell(g) = R(g)/g^p$ gives $\ell(\lambda g)/\ell(g) \rightarrow 1$ [8]. $\square$

Corollary 5.3 (Pure power as an exact fixed point). *If $R(\lambda g) = \lambda^p R(g)$ for every $g, \lambda > 0$, then*

$$R(g) = t g^p$$

for $t = R(1)$. Conversely, every pure power has exact scale covariance.

The exact power is therefore one fixed point. Stable dilation limits also admit a slowly varying envelope, which can remain numerically important over every realizable design scale. The theorem separates two pieces of the profile:

$$\log R(g) = p \log g + \log \ell(g).$$

The first term is the scale index; the second is the finite-scale envelope. A fitted line estimates both at once. CRG therefore treats a measured exponent as a windowed observation of a typed realization profile rather than as an intrinsic scalar attached to the computation.

5.1 Preferred dilation factors

Digital hierarchies often repeat only at a declared factor $b > 1$. Covariance under one factor is weaker than regular variation.

Theorem 5.4 (Discrete scale covariance). *Fix $b > 1$. A positive function satisfies*

$$R(bg) = b^p R(g) \quad \text{for all } g > 0$$

if and only if

$$\boxed{R(g) = g^p \psi(\log_b g)}$$

for a positive 1-periodic function ψ .

Proof. Define $\psi(t) = b^{-pt} R(b^t)$. The covariance relation gives $\psi(t+1) = \psi(t)$. Substitution proves the converse. $\square$

This is the discrete-scale-invariance mechanism behind log-periodic corrections [9]. A binary or quadtree hierarchy can have a stable mean exponent while retaining a phase oscillation in logarithmic coordinates.

Theorem 5.5 (Two-scale criterion). *Let R be eventually nondecreasing. Suppose there exist $a, b > 1$ and $p \geq 0$ such that $\log a / \log b$ is irrational and*

$$\frac{R(ag)}{R(g)} \rightarrow a^p, \quad \frac{R(bg)}{R(g)} \rightarrow b^p.$$

Then $R \in \text{RV}_p$.

Proof. The subgroup $\{a^m b^n : m, n \in \mathbb{Z}\}$ is dense in $\mathbb{R}_+$. Repeated application of the two limits, including reciprocal limits for negative powers, gives $R(hg)/R(g) \rightarrow h^p$ for every subgroup element h . For arbitrary $\lambda > 0$, choose subgroup elements $h_- < \lambda < h_+$ arbitrarily close to λ . Eventual monotonicity gives

$$\frac{R(h_-g)}{R(g)} \leq \frac{R(\lambda g)}{R(g)} \leq \frac{R(h_+g)}{R(g)}.$$

First take $g \rightarrow \infty$ and then $h_{\pm} \rightarrow \lambda$. □

One preferred factor can therefore preserve hierarchy phase; stable scaling at two incommensurate factors removes that ambiguity. In physical terms, a binary hierarchy can revisit the same structural phase every time scale is multiplied by two. A second incommensurate dilation probes the profile at a dense set of logarithmic phases and forces the periodic factor to collapse to a constant.

5.2 Normalized dilation action and phase ratios

Definition 5.6 (Normalized dilation action). *For a positive measurable profile, considered modulo a positive amplitude, choose the representative $R(1) = 1$ and define*

$$(\mathcal{D}_a R)(x) = \frac{R(ax)}{R(a)}, \quad a > 0.$$

Then $\mathcal{D}_a \mathcal{D}_c = \mathcal{D}_{ac}$.

Proposition 5.7 (Fixed points and scale cycles). *A positive measurable profile is fixed by every $\mathcal{D}_a$ exactly when $R(x) = x^p$. For a declared hierarchy factor $b > 1$, it is fixed by $\mathcal{D}_b^h$ exactly when*

$$R(x) = x^p \psi(\log_b x)$$

for a positive h -periodic ψ normalized by $\psi(0) = 1$. Finally, $\mathcal{D}_a R \rightarrow x^p$ locally uniformly as $a \rightarrow \infty$ exactly when $R \in \text{RV}_p$.

Proof. Full invariance is the measurable multiplicative equation $R(ax) = R(a)R(x)$, whose positive solutions are powers. Discrete invariance follows by setting $\psi(t) = b^{-pt} R(b^t)$ and imposing h -periodicity; the converse is immediate. The last statement is the uniform convergence theorem for regular variation [8]. □

The following result characterizes the observable scalar sequence. It does not infer the latent mechanism that generated that sequence.

Theorem 5.8 (Phase-ratio characterization). *Fix $g_k = g_0 b^k$ and a positive sequence Y_k with its canonical log-linear interpolation. For an integer $h \geq 1$, the phasewise limits*

$$\frac{Y_{nh+j+1}}{Y_{nh+j}} \rightarrow c_j > 0, \quad j = 0, \dots, h-1,$$

exist if and only if

$$Y(g) = Y(g_0)(g/g_0)^p L(g) \psi(\log_b(g/g_0)),$$

where

$$p = \frac{\log \prod_{j=0}^{h-1} c_j}{h \log b},$$

$L \in \text{RV}_0$, and ψ is positive, log-linearly interpolated, and h -periodic. Under this interpolation, $Y \in \text{RV}_p$ exactly when all $c_j = b^p$. If

$$\sum_k \left| \log \frac{Y_{k+1}/Y_k}{c_{k \bmod h}} \right| < \infty,$$

then $L(g) \rightarrow L_\infty \in (0, \infty)$.

Proof. Set $a_0 = 1$ and $a_{j+1} = a_j c_j b^{-p}$; the definition of p gives $a_h = a_0$. For $k = nh + j$, define

$$L_k = \frac{Y_k}{Y_0 b^{pk} a_j}.$$

Then $L_{k+1}/L_k \rightarrow 1$. An interval from g to λg crosses only a bounded number of hierarchy cells, and the logarithmic increment on each late cell tends to zero. Hence the log-linear interpolation of L_k is slowly varying. Periodically interpolate a_j to obtain ψ . Substitution proves the reverse implication. A nonconstant ψ leaves distinct phase-subsequence ratios, whereas absolute summability makes $\log L_k$ a convergent series. $\square$

The adjacent log slopes

$$s_k = \frac{\log(Y_{k+1}/Y_k)}{\log b}$$

therefore converge to one value in the regularly varying class, converge to an h -cycle in the finite-phase class, and do neither outside those output classes. The theorem provides an observable classification independent of a latent mechanism. It answers the empirical question first—what ratio law does the measured profile exhibit?—and leaves the mechanistic question to the positive-mode and mixture theorems that follow.

6 Windowed estimators and finite-scale bias

For a differentiable profile define the local logarithmic slope

$$p_{\text{loc}}(g) = \frac{d \log R(g)}{d \log g}.$$

If $R(g) = g^p \ell(g)$, then

$$p_{\text{loc}}(g) = p + \frac{d \log \ell(g)}{d \log g}.$$

The correction need not be monotone and need not be small on realizable scales.

Theorem 6.1 (Multiplicative-window convergence). *Let $R(g) = g^p \ell(g)$ with ℓ slowly varying. Fix distinct multipliers $\lambda_1, \dots, \lambda_J > 0$. At anchor t , fit the unweighted ordinary-least-squares slope of*

$$(\log(t\lambda_j), \log R(t\lambda_j))_{j=1}^J.$$

Then

$$\boxed{\widehat{p}(t) \rightarrow p.}$$

Proof. Let a_j be the OLS slope weights. They are independent of t and satisfy

$$\sum_j a_j = 0, \quad \sum_j a_j \log \lambda_j = 1.$$

Therefore

$$\widehat{p}(t) = p + \sum_j a_j \log \ell(t\lambda_j) = p + \sum_j a_j \log \frac{\ell(t\lambda_j)}{\ell(t)}.$$

Every term tends to zero by slow variation. $\square$

A fitted slope can thus converge under outward shifts of a fixed multiplicative window even when the exact relation is not a pure power. The slowly varying term controls finite-window bias. For example, $R(g) \asymp g/\log g$ belongs to RV_1 but has smooth local slope

$$1 - \frac{1}{\log g},$$

which converges slowly. Butterfly and FFT-like recursive structures provide canonical examples of this correction [16].

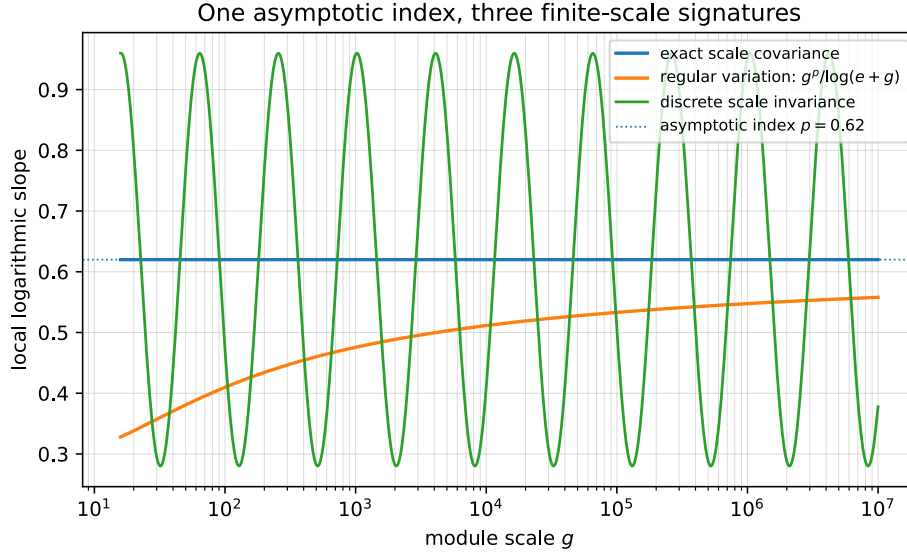


Figure 1: Exact scale covariance, regular variation with a logarithmic correction, and discrete scale invariance can share an asymptotic index while producing different finite-scale slopes.

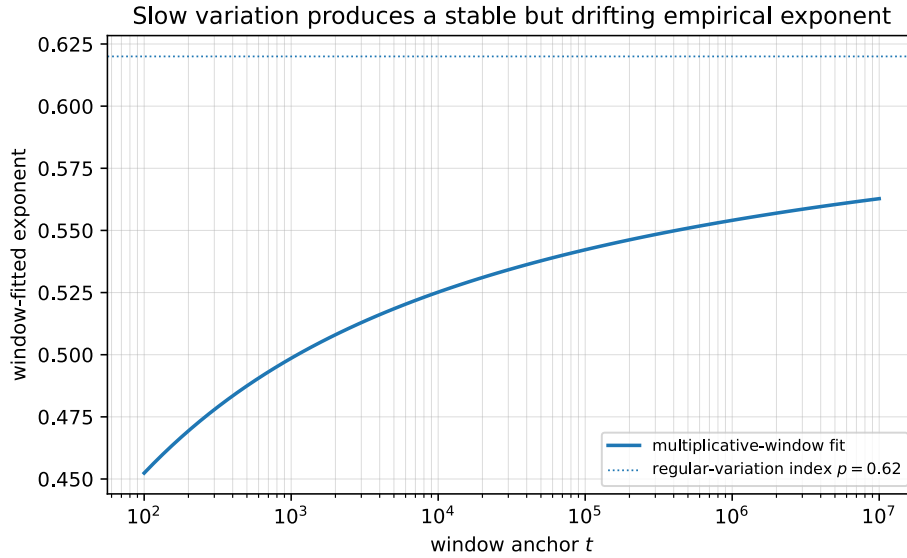


Figure 2: A multiplicative-window regression of $g^{0.62} / \log(e + g)$ converges to its regular-variation index while retaining finite-scale bias.

7 Recursive mechanisms and certified scaling basins

The output characterizations in [Section 5](#) identify the forms of stable scalar profiles. This section derives mechanisms that produce those forms from recursive realization structure. A hierarchy level is not represented by one number alone: its boundary traffic can be decomposed into modes associated with components, interfaces, fanout classes, protocol classes, or other typed contributions. The vector v_k records those modes; the matrix A_k records how one level composes them into the next; and the observation vector u selects the profile reported to the designer.

Recursive positive composition supplies a renormalization mechanism. Projective mixing forgets the initial mode proportions while retaining total amplitude; periodic composition retains a hierarchy phase; vanishing perturbations

create a slowly varying envelope. This yields a derivation of scale behavior from a registered realization mechanism rather than an inference from a fitted line.

Theorem 7.1 (Summable scalar distortion). *Let $b > 1$, $c > 0$, and suppose a positive profile on $g_k = g_0 b^k$ obeys*

$$R(g_{k+1}) = cR(g_k)(1 + \varepsilon_k), \quad \varepsilon_k > -1,$$

with $\sum_{k \geq 0} |\varepsilon_k| < \infty$. Then $\prod_{j < k} (1 + \varepsilon_j)$ converges to a finite positive limit and

$$R(g_k) = \Theta(c^k) = \Theta(g_k^p), \quad p = \log_b c.$$

Under the canonical log-linear interpolation, the profile is regularly varying with index p and in fact has an asymptotically constant reduced amplitude.

Proof. Iteration gives

$$R(g_k) = R(g_0) c^k \prod_{j < k} (1 + \varepsilon_j).$$

Absolute summability, positivity of every factor, and the convergence of $\sum_j \log(1 + \varepsilon_j)$ after a finite prefix imply a positive finite product limit. Since $c^k = (g_k/g_0)^{\log_b c}$, the result follows. $\square$

The vector mechanism retains mode composition and hierarchy phase rather than reducing the recursion to one scalar multiplier.

Definition 7.2 (Positive interface-mode cocycle). *At scales $g_k = g_0 b^k$, let $v_k \in \mathbb{R}_{++}^m$ collect typed interface modes, let $u \in \mathbb{R}_{++}^m$ be the declared observation vector, and write*

$$v_{k+1} = A_k v_k + \eta_k, \quad Y_k = u^\top v_k, \quad z_k = \frac{v_k}{u^\top v_k}.$$

The relative recursion error is

$$\delta_k = \max_i \frac{|(\eta_k)_i|}{(A_k v_k)_i}.$$

Every sufficiently late A_k is nonnegative and maps the cone interior into itself, and each perturbed update remains in the cone interior. The law is asymptotically h -periodic when

$$A_{nh+j} \longrightarrow A^{(j)}, \quad j = 0, \dots, h-1,$$

in any fixed matrix norm. Each limiting phase map preserves the cone interior. With

$$M = A^{(h-1)} \dots A^{(0)},$$

the limiting law is projectively mixing after finite coarse graining when $M^\nu > 0$ entrywise for some fixed integer $\nu \geq 1$.

For $x, y > 0$, Hilbert's projective metric is

$$d_H(x, y) = \log \frac{\max_i x_i / y_i}{\min_i x_i / y_i}.$$

A positive matrix has finite projective diameter and contracts this metric [10, 11].

Theorem 7.3 (Perron–Floquet scaling on the certified mixing class). *Under Theorem 7.2, suppose $v_0 > 0$, $M^\nu > 0$ for some fixed ν , and $\delta_k \rightarrow 0$. Then:*

(i) there is a unique positive u -normalized phase cycle $r_0, \dots, r_{h-1}$ such that

$$z_{nh+j} \rightarrow r_j, \quad r_{j+1} = \frac{A^{(j)} r_j}{u^\top A^{(j)} r_j},$$

for every admitted positive initial trajectory, with phase indices read modulo h ;

(ii) with $c_j = u^\top A^{(j)} r_j$,

$$\frac{Y_{nh+j+1}}{Y_{nh+j}} \rightarrow c_j, \quad \prod_{j=0}^{h-1} c_j = \text{spr}(M);$$

(iii) the profile has

$$Y(g) = Y(g_0)(g/g_0)^p L(g) \psi(\log_b(g/g_0)), \quad p = \frac{\log \text{spr}(M)}{h \log b},$$

where $L \in \text{RV}_0$ and ψ is positive and h -periodic; and

(iv) at hierarchy phase j ,

$$\mathcal{D}_{g_0 b^{nh+j}} Y(x) \rightarrow x^p \frac{\psi(j + \log_b x)}{\psi(j)}$$

locally uniformly for $x > 0$.

If all $c_j = b^p$, then $Y \in \text{RV}_p$. A nonconstant minimal phase sequence yields asymptotic discrete scale invariance. If

$$\sum_k \left(\|A_k - A^{(k \bmod h)}\| + \delta_k \right) < \infty,$$

then $L(g) \rightarrow L_\infty \in (0, \infty)$.

Proof. Write $[x]_u = x/(u^\top x)$. On the normalized cone section $\{z > 0 : u^\top z = 1\}$, define the unperturbed maps

$$\mathcal{F}_k(z) = \frac{A_k z}{u^\top A_k z}.$$

Because $\delta_k \rightarrow 0$, eventually $\delta_k < 1$. The componentwise relative error bound gives

$$(1 - \delta_k) A_k v_k \leq A_k v_k + \eta_k \leq (1 + \delta_k) A_k v_k.$$

Consequently,

$$d_H(z_{k+1}, \mathcal{F}_k(z_k)) \leq \log \frac{1 + \delta_k}{1 - \delta_k} = O(\delta_k). \quad (7.1)$$

This is the required perturbation bookkeeping. It permits signed η_k provided the relative bound and cone-interior condition hold.

Group the recursion into periods and then into primitive blocks:

$$B_n = A_{nh+h-1} \cdots A_{nh}, \quad C_n = B_{n\nu+\nu-1} \cdots B_{n\nu}.$$

Then $B_n \rightarrow M$ and $C_n \rightarrow M^\nu > 0$ entrywise. After a finite prefix, the νh -step phase-zero maps have uniformly bounded projective diameter and one Birkhoff contraction coefficient $\kappa_{\text{ctr}} < 1$. Let r_0 be the u -normalized Perron direction of M , equivalently of M^ν , and set $e_n = d_H(z_{n\nu h}, r_0)$. Nonnegative cone-interior maps are nonexpansive in Hilbert metric. Applying (7.1) through one finite block, using $C_n \rightarrow M^\nu$ and $[M^\nu r_0]_u = r_0$, gives

$$e_{n+1} \leq \kappa_{\text{ctr}} e_n + \epsilon_n, \quad \epsilon_n \rightarrow 0.$$

Hence $e_n \rightarrow 0$. Propagation through the intervening period and phase maps gives

$$z_{nh+j} \rightarrow r_j, \quad r_{j+1} = [A^{(j)} r_j]_u.$$

Because $[M r_0]_u = r_0$, the cycle closes. Primitivity of M makes r_0 , and therefore the whole phase cycle, unique and independent of v_0 .

The scalar gain satisfies

$$\frac{Y_{k+1}}{Y_k} = u^\top A_k z_k + \frac{u^\top \eta_k}{Y_k}. \quad (7.2)$$

The perturbation term is controlled without assuming cancellation:

$$\left| \frac{u^\top \eta_k}{Y_k} \right| \leq \delta_k u^\top A_k z_k = o(1). \quad (7.3)$$

Matrix and state convergence therefore yield the phasewise gains $c_j > 0$. The cycle relation is $A^{(j)}r_j = c_j r_{j+1}$; multiplying over one period gives

$$Mr_0 = \left(\prod_j c_j \right) r_0.$$

Perron–Frobenius theory implies $\prod_j c_j = \text{spr}(M)$. The profile representation and dilation limits now follow from [Theorems 5.7](#) and [5.8](#).

For the strengthened conclusion, discard a finite prefix. The normalized trajectory and limiting cycle then lie in a compact subset of the cone interior, where normalization and the positive gain functional are uniformly Lipschitz. If β_n is the sum of matrix and relative errors in the n th νh -block, summability sharpens the recurrence to

$$e_{n+1} \leq \kappa_{\text{ctr}} e_n + C\beta_n, \quad \sum_n \beta_n < \infty.$$

The contracted recurrence gives $\sum_n e_n < \infty$; finite-step propagation then gives

$$\sum_k d_H(z_k, r_{k \bmod h}) < \infty.$$

Equations (7.2)–(7.3) and Lipschitz continuity imply

$$\sum_k \left| \log \frac{Y_{k+1}/Y_k}{c_{k \bmod h}} \right| < \infty.$$

The last clause of [Theorem 5.8](#) makes the slowly varying envelope converge to a positive finite constant. $\square$

Interpretation of the Perron–Floquet exponent. Over one complete hierarchy period, the limiting product M maps a mode mixture back to the same projective phase and multiplies its amplitude by $\text{spr}(M)$. The hierarchy scale grows by b^h during the same period. Equating these two multiplicative changes gives

$$(b^h)^p = \text{spr}(M), \quad p = \frac{\log \text{spr}(M)}{h \log b}.$$

The exponent is therefore the ratio of amplitude growth to scale growth. The phase factors c_j distribute that period gain among individual hierarchy steps, and the slowly varying factor records the residual nonperiodic drift.

Corollary 7.4 (Stationary primitive recursion). *If $A_k \rightarrow A$, A is primitive, and the relative recursion error tends to zero, then*

$$\frac{v_k}{u^\top v_k} \rightarrow r, \quad \frac{Y_{k+1}}{Y_k} \rightarrow \text{spr}(A), \quad Y \in \text{RV}_{\log_b \text{spr}(A)}.$$

If the matrix and relative errors are summable, then $Y(g) \sim Cg^{\log_b \text{spr}(A)}$.

Proof. Apply [Theorem 7.3](#) with $h = 1$. $\square$

Remark 7.5 (Scope of the basin certificate). *The theorem classifies every admitted trajectory in the asymptotically finite-periodic, projectively mixing, finite-mode class with vanishing relative error. It is an in-basin result after membership hypotheses are supplied. It does not say that positivity, finite mode dimension, projective mixing, or periodicity is necessary for an observed regularly varying profile. The scalar equivalence in [Theorem 5.8](#) characterizes output ratios, not latent dynamics. Nonlinear, infinite-mode, stochastic, imprimitive, quasiperiodic, or nonpositive mechanisms remain outside this certificate and may require different theorems.*

8 Arithmetic modes and finite-scale curvature

For a fixed realization and hierarchy, classify boundary events into disjoint internal component classes and an interface class. Additive traffic accounting gives

$$R(g) = \sum_{j=1}^J R_j(g) + R_\partial(g). \tag{8.1}$$

For a netlist hypergraph $H = (V, E)$ and module partition Π_{part} , let

$$\mathcal{C}_H(\Pi_{\text{part}}) = \sum_{M \in \Pi_{\text{part}}} T_H(M)$$

be aggregate terminal incidence. If V is divided into disjoint components V_i , with internal hyperedge sets E_i and interface hyperedges E_∂ , then

$$\mathcal{C}_H(\Pi_{\text{part}}) = \sum_i \mathcal{C}_{H_i}(\Pi_{\text{part}}|_{V_i}) + \mathcal{C}_\partial(\Pi_{\text{part}}).$$

Dividing by $|\Pi_{\text{part}}|$ proves (8.1). Under the sum-of-external-degrees convention, every cut hyperedge contributes once to every module it touches [12, 13]. The profile therefore composes arithmetically; fitted exponents generally do not. In the differential identities below, the interface profile is included as one of the indexed positive modes whenever it is active.

Let $x = \log g$ and let each positive mode $R_j(e^x)$ be twice differentiable. Define its local slope and traffic share by

$$p_j^{\text{loc}}(g) = \frac{d \log R_j(g)}{d \log g}, \quad \omega_j(g) = \frac{R_j(g)}{\sum_k R_k(g)}.$$

The additive identity permits an exact differential decomposition. Writing $x = \log g$, each mode obeys $R'_j(x) = p_j^{\text{loc}}(x) R_j(x)$ and the total profile is $R = \sum_j R_j$. The traffic share $\omega_j = R_j/R$ therefore acts as a probability weight over active modes. Differentiating once gives the weighted mean of local exponents; differentiating again separates curvature created by differences among modes from curvature already present within each mode.

Theorem 8.1 (Mode-mixture geometry). *For $R(g) = \sum_j R_j(g)$,*

$$p_{\text{loc}}(g) = \sum_j \omega_j(g) p_j^{\text{loc}}(g),$$

and

$$\frac{dp_{\text{loc}}}{d \log g} = \text{Var}_{\omega(g)}(p_j^{\text{loc}}(g)) + \sum_j \omega_j(g) \frac{dp_j^{\text{loc}}(g)}{d \log g}.$$

Proof. Differentiate $\log R$ to obtain the first identity. With $x = \log g$,

$$\omega'_j = \omega_j(p_j^{\text{loc}} - p_{\text{loc}}).$$

Differentiating $p_{\text{loc}} = \sum_j \omega_j p_j^{\text{loc}}$ and collecting terms gives the variance and drift decomposition. $\square$

For pure modes $R_j = a_j g^{p_j}$, the within-mode drift vanishes:

$$p_{\text{loc}} = \sum_j \omega_j p_j, \quad \frac{dp_{\text{loc}}}{d \log g} = \text{Var}_{\omega}(p_j) \geq 0.$$

Finite-scale curvature is thus a structural observable of heterogeneous crossover, not merely regression noise.

Corollary 8.2 (No exact heterogeneous scalar). *A positive finite sum $\sum_j a_j g^{p_j}$ equals one power law on an open interval if and only if all active exponents p_j are equal.*

Proof. One power has zero log-curvature. By Theorem 8.1, pure modes have zero curvature exactly when their weighted exponent variance is zero. $\square$

Theorem 8.3 (Dominant regularly varying mode). *Let $R_j \in \text{RV}_{p_j}$ be a finite collection of positive functions and let $p_\star = \max_j p_j$. Then*

$$\sum_j R_j \in \text{RV}_{p_\star}.$$

Every mode with exponent below $p_\star$ becomes asymptotically negligible relative to a positive mode with exponent $p_\star$.

Proof. Write $R_j(g) = g^{p_j} \ell_j(g)$. Ratios of regularly varying functions with distinct indices satisfy $R_j/R_k \rightarrow 0$ when $p_j < p_k$ [8]. A finite sum of positive slowly varying functions is slowly varying; factor out $g^{p_\star}$. $\square$

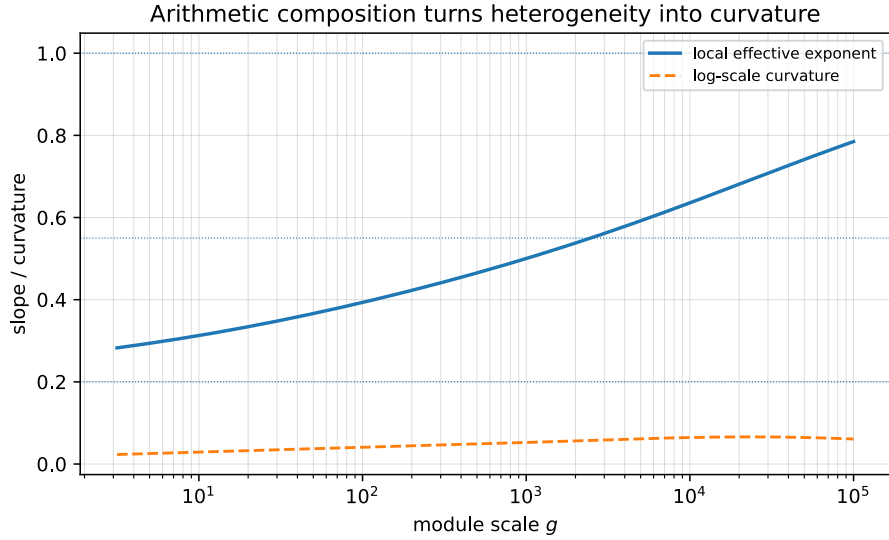


Figure 3: For pure modes, the whole-profile local slope is a traffic-weighted convex mixture and its log-curvature is the weighted exponent variance.

9 Interface reach and terminal-boundary crossover

Let E_∂ be the inter-component hyperedge set. At a size- g hierarchy level, let $\kappa_g(e)$ be the number of modules touched by hyperedge e . Under sum-of-external-degrees accounting, the exact interface profile is

$$R_\partial(g) = \frac{g}{G} \sum_{e \in E_\partial} \kappa_g(e) \mathbf{1}[\kappa_g(e) \geq 2]. \quad (9.1)$$

Interface-net count alone is not the multiscale variable; pin occupancy and hierarchy reach are part of the accounting.

For a two-pin net, $\kappa_g(e) = 2$ exactly while its endpoints remain separated. If $N_\partial^\geq(g)$ counts such nets, then

$$R_\partial(g) = \frac{2g}{G} N_\partial^\geq(g).$$

If $N_\partial^\geq(g) \asymp G^\alpha g^{-\vartheta} \ell(g)$, the interface mode has index $p_\partial = 1 - \vartheta$ up to the slowly varying term. Against a core $R_{\text{core}}(g) = \Theta(g^p)$ at $g = \Theta(G^\beta)$, it is relevant when

$$\alpha > 1 - \beta(p_\partial - p).$$

Uniform random two-pin endpoints give $p_\partial = 1$ away from terminal saturation.

For an r -pin hyperedge whose pins independently choose among $m = G/g$ equal modules,

$$\mathbb{E}[\kappa_g(e)] = m \left[1 - \left(1 - \frac{1}{m} \right)^r \right],$$

and all pins occupy one module with probability m^{1-r} . Hence

$$\mathbb{E}[\kappa_g(e) \mathbf{1}(\kappa_g(e) \geq 2)] = m \left[1 - \left(1 - \frac{1}{m} \right)^r \right] - m^{1-r}.$$

This is a fanout-resolved null model, not a placement model. A global broadcast hyperedge instead touches every module and contributes one unit to the level mean. If $K = \Theta(G^\alpha)$ global nets are present, then

$$R_{\text{broadcast}}(g) = \Theta(G^\alpha),$$

which dominates a core at $g = \Theta(G^\beta)$ when

$$\alpha > \beta p.$$

Clocks, resets, and coherence fabrics must therefore retain their actual fanout profiles rather than use a two-pin surrogate.

Definition 9.1 (Declared cap response). *A cap response is a continuous nondecreasing function $\Phi : (0, \infty) \rightarrow (0, 1]$ satisfying*

$$\Phi(z)/z \rightarrow 1 \quad (z \downarrow 0), \quad \Phi(z) \rightarrow 1 \quad (z \rightarrow \infty).$$

For an interior profile I and declared terminal budget B , define

$$\mathcal{C}_B[I](g) = B\Phi(I(g)/B).$$

The hard cap is $\Phi(z) = \min\{z, 1\}$; a soft rational cap is $\Phi(z) = z/(1+z)$.

Theorem 9.2 (Boundary crossover). *Let $p > 0$, let $B_n \rightarrow \infty$, and choose $g_n \rightarrow \infty$ with $I(g_n)/B_n \rightarrow 1$.*

(i) *If $I \in \text{RV}_p$, then*

$$\frac{\mathcal{C}_{B_n}[I](g_n x)}{B_n} \longrightarrow \Phi(x^p)$$

locally uniformly for $x > 0$. The hard-cap limit is $\min\{x^p, 1\}$.

(ii) *If instead*

$$I(g) = g^p L(g) \psi(\log_b g),$$

where $L \in \text{RV}_0$ and ψ is positive, continuous, and h -periodic, then every subsequence satisfying $\log_b g_n \bmod h \rightarrow s$ has the phase-indexed limit

$$\frac{\mathcal{C}_{B_n}[I](g_n x)}{B_n} \longrightarrow \Phi\left(x^p \frac{\psi(s + \log_b x)}{\psi(s)}\right).$$

Proof. Uniform convergence for regular variation gives $I(g_n x)/I(g_n) \rightarrow x^p$ locally uniformly. Combine it with $I(g_n)/B_n \rightarrow 1$ and continuity of Φ . In the finite-phase branch, slow variation cancels while the periodic factor retains the limiting hierarchy phase. $\square$

Corollary 9.3 (Crossover slope). *In the regularly varying branch of Theorem 9.2, additionally suppose*

$$\frac{gI'(g)}{I(g)} \rightarrow p$$

and $\Phi \in C^1$ near x^p . Then the local logarithmic slope at $g_n x$ converges to

$$pE_\Phi(x^p), \quad E_\Phi(z) = \frac{z\Phi'(z)}{\Phi(z)}.$$

For the hard cap, the statement holds away from $x = 1$, with one-sided slopes at the kink.

Theorem 9.4 (Partial converse from phase-rich cap collapse). *Fix $p > 0$ and let $B_n > 0$. Let I be positive and eventually nondecreasing, let $g_n \uparrow \infty$ satisfy $g_{n+1}/g_n \rightarrow 1$, and suppose $I(g_n)/B_n \rightarrow 1$. Let J be an open multiplier interval containing 1. Assume Φ restricts to a homeomorphism on an interval containing $\{x^p : x \in J\}$ and all sufficiently late normalized arguments. If*

$$\frac{\mathcal{C}_{B_n}[I](g_n x)}{B_n} \longrightarrow \Phi(x^p)$$

locally uniformly for $x \in J$, then $I \in \text{RV}_p$.

Proof. Continuity of Φ^{-1} gives

$$\frac{I(g_n x)}{B_n} \rightarrow x^p, \quad \frac{I(g_n x)}{I(g_n)} \rightarrow x^p$$

locally uniformly on J . For arbitrary $t \rightarrow \infty$, bracket t between adjacent g_n, g_{n+1} . Eventual monotonicity, the dense-anchor condition, and slightly smaller and larger multipliers in J squeeze $I(tx)/I(t)$ to x^p for $x \in J$. Finite products and reciprocals of an open interval containing 1 cover $(0, \infty)$, so the dilation limit holds for every positive multiplier. $\square$

The cap shape is a declared model component represented by Φ . A purely dyadic grid samples a period-one function of $\log_2 g$ at one phase and cannot meet the dense-anchor premise of the converse. The two-sided converse also excludes the hard cap because it is noninjective after saturation; a one-sided version can use an unsaturated interval. Terminal budgets and pinned boundary conditions are coordinates of the resource contract ρ , not measurement noise.

10 Finite-scale observability and failure taxonomy

At one level, let per-module traffic be a positive random variable T . Jensen's inequality gives

$$\log \mathbb{E}[T] - \mathbb{E}[\log T] \geq 0.$$

Thus the logarithm of the level mean and the mean of per-module logarithms are different estimators even before fitting across scales. For $\text{CV}(T) \ll 1$ and the usual moment regularity,

$$\log \mathbb{E}[T] - \mathbb{E}[\log T] = \frac{1}{2} \text{CV}(T)^2 + o(\text{CV}(T)^2).$$

Heavy-tailed module traffic can make the discrepancy large.

Repeated composition can also contract variation around an expected profile. Fix levels $g_1, \dots, g_J$ and write

$$Y_j = \sum_{r=1}^N Z_{rj} > 0,$$

where the contributor vectors Z_r are independent, although coordinates within one vector may be arbitrarily dependent across levels. Suppose Z_{rj} lies in an interval of width Δ_{rj} . For $x_j = \log g_j$, let a_j be the OLS slope weights, so that

$$\hat{p} = \sum_{j=1}^J a_j \log Y_j.$$

Theorem 10.1 (Concentration of a fitted slope). *Let*

$$\mu_j = \mathbb{E}[Y_j] \geq \mu_{\min} > 0, \quad V = \max_j \sum_{r=1}^N \Delta_{rj}^2,$$

and define

$$p_\star = \sum_{j=1}^J a_j \log \mu_j.$$

For $0 < \delta < 1$, set

$$t = \sqrt{\frac{V}{2} \log \frac{2J}{\delta}}.$$

If $t \leq \mu_{\min}/2$, then with probability at least $1 - \delta$,

$$|\hat{p} - p_\star| \leq \frac{2\|a\|_1}{\mu_{\min}} \sqrt{\frac{V}{2} \log \frac{2J}{\delta}}.$$

Proof. Hoeffding's inequality and a union bound control every $|Y_j - \mu_j|$ by t . On that event $Y_j \geq \mu_{\min}/2$, and the mean-value theorem gives

$$|\log Y_j - \log \mu_j| \leq \frac{2}{\mu_{\min}} |Y_j - \mu_j|.$$

Apply the triangle inequality to the linear slope functional. $\square$

For comparable contributors averaged with weights $1/m$, the width proxy is $V = O(1/m)$. For unequal normalized contribution weights α_i , the corresponding effective count is

$$m_{\text{eff}} = \frac{1}{\sum_i \alpha_i^2}.$$

Repeated IP, common clocks, and shared coherence protocols can make m_{eff} far smaller than the raw block count.

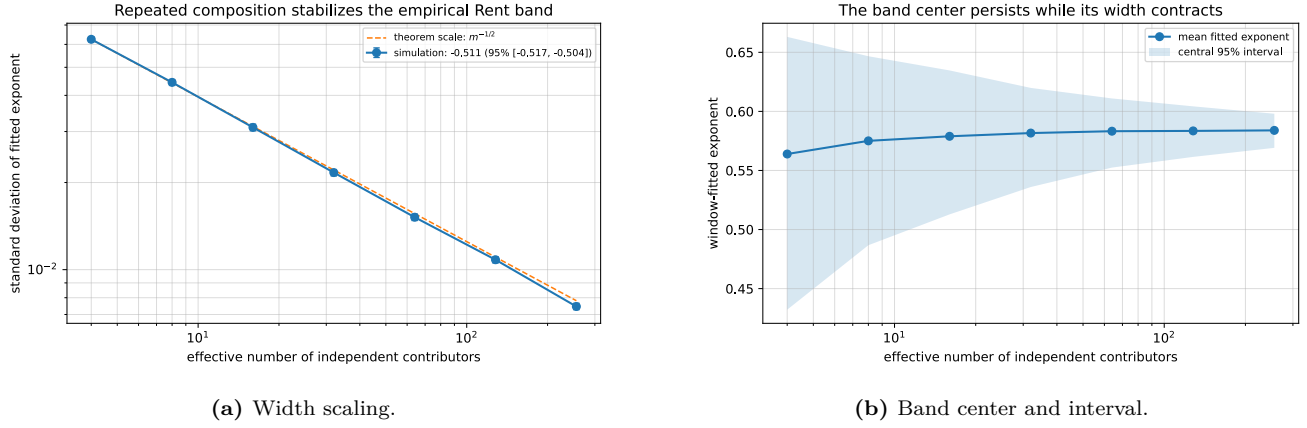


Figure 4: Under the bounded-independent composition model, a persistent expected center coexists with a shrinking distribution of fitted slopes.

Dynamical hypothesis	Profile consequence	Observable diagnostic	Failure meaning
Autonomous mixing cocycle	$g^p L(g)$	Mode direction and adjacent slope converge	Rejects this certificate, not all scaling mechanisms
Summable autonomous distortion	$Cg^p(1 + o(1))$	Reduced amplitude converges	Rejects summable correction
h -periodic mixing cocycle	$g^p L(g)\psi(\log_b g)$	Phase slopes converge to an h -cycle	Rejects the finite-period certificate
Regular interior plus declared cap	$B\Phi((g/g_*)^p)$	Normalized crossover collapse	Rejects the registered cap model
Finite-phase interior plus cap	Phase-indexed cap curves	Collapse depends on log phase	One-phase samples cannot certify membership
No convergent or periodic gain law	No class asserted	Adjacent ratios neither converge nor cycle	Outside this sufficient taxonomy

Table 1: Mechanism, consequence, and falsifier are separated. Observing a consequence does not prove latent basin membership.

Theorem 10.2 (Stable empirical generalized law). *Let the expected profile be $\mu(g) = g^p \ell(g)$ with ℓ slowly varying, and measure on a fixed multiplicative window $g_j(t) = t\lambda_j$. Suppose the independent bounded-composition conditions of Theorem 10.1 hold and its stochastic bound is $O(m_{\text{eff}}^{-1/2})$. Then*

$$\hat{p}(t, m_{\text{eff}}) = p + b_\ell(t) + O_{\mathbb{P}}(m_{\text{eff}}^{-1/2}),$$

where

$$b_\ell(t) = \sum_j a_j \log \frac{\ell(t\lambda_j)}{\ell(t)} \rightarrow 0.$$

Consequently, $\hat{p} \rightarrow p$ in probability when the window moves outward and $m_{\text{eff}} \rightarrow \infty$.

Proof. The expected-profile slope is $p + b_\ell(t)$ by Theorem 6.1; the stochastic deviation follows from Theorem 10.1. $\square$

This is a conditional mechanism for an empirical band: slow variation controls its finite-window bias, mode crossover controls curvature, and effective independent composition controls width. It does not identify the mechanism behind a historical fitted band or make its center universal.

Trees, lattices, expanders, butterflies, and heterogeneous systems calibrate different rows of this taxonomy: bounded tree profiles can have index 0; d -dimensional lattices have surface-growth index $(d-1)/d$; expanders have linear boundary growth; butterfly networks carry an index-one $1/\log g$ correction; and heterogeneous systems combine several modes with an explicit interface profile. These examples calibrate distinct scaling classes; assigning a computation to a class requires the corresponding structural hypotheses. At all realizable sizes, the measured profile, hierarchy convention, correction term, and estimator remain primary; an asymptotic index can be operationally remote.

10.1 Deterministic mechanism checks

The phase-rich deterministic check uses the declared scalar observation vector $u = (1, 1)^\top$ and

$$A^{(0)} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}, \quad A^{(1)} = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}.$$

For $M = A^{(1)}A^{(0)}$, $\text{spr}(M) = 4 + \sqrt{15}$ and

$$p = \frac{\log(4 + \sqrt{15})}{2 \log 2} \approx 1.488455.$$

The reference script `artifacts/renormalization_validation_v1.py` evaluates every level $k = 0, \dots, 64$ for four positive initial mixtures. The recorded phase gains are $c_0 = 2.4364916731$ and $c_1 = 3.2312785769$, while $2^p = 2.8058837015$ differs from both. Across late samples, the maximum Hilbert distance to the phase cycle is 4.44×10^{-16} , the maximum adjacent-ratio error is 8.88×10^{-16} , and the relative tail span of the phase-reduced amplitude is 8.61×10^{-15} . The product $c_0 c_1$ agrees with $4 + \sqrt{15}$ to recorded precision.

The same program registers quasiperiodic, nonnegligibly forced, nonmixing, and bounded-no-exponent controls under `artifacts/renormalization_results_v1/`. These computations verify the bookkeeping and observables. The proofs establish the theorem statements, while physical-netlist basin membership is determined by the registered structural hypotheses and diagnostics.

11 Public-netlist diagnostics and mode-resolved attribution

The scaling theory distinguishes three questions: the output law of a profile, a mechanism that can generate that law, and a measurement instrument that observes a finite realization. This section addresses the third. Two deterministic hierarchy instruments instantiate reproducible, hierarchy-conditioned scale profiles and test finite-scale identities, estimator dependence, and cross-hierarchy transfer.

The diagnostics use ten named BLIF snapshots from the public EPFL combinational benchmark suite [15], ranging from 374 to 57,503 vertices. Every reported value belongs to a complete measurement tuple

(netlist, parser, hierarchy, traffic convention, scale window, estimator).

This tuple is the empirical object. It permits reproducible comparison of hierarchy and estimator effects while keeping the logical netlist fixed.

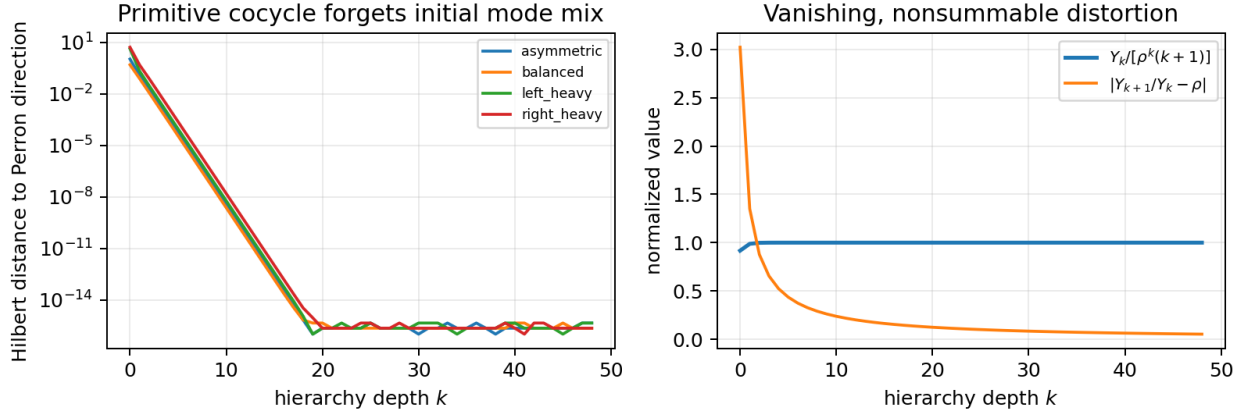
Two deterministic hierarchy instruments are used. The first computes one reverse-Cuthill–McKee (RCM) ordering and divides it into contiguous, size-regular, power-of-two module counts. It is run on all ten files. This instrument is geography-like but is not cut optimized. The second is a recursive RCM hierarchy: each module above a declared threshold is reordered locally before bisection, while inherited order is preserved below the threshold. It is run on the barrel shifter and multiplier. Bidirectional cross-hierarchy transfer is evaluated only on the multiplier. The paired runs vary hierarchy construction while holding parser and traffic accounting fixed.

11.1 Registered measurement and reproduction protocol

The measurement contract is registered at the following resolution.

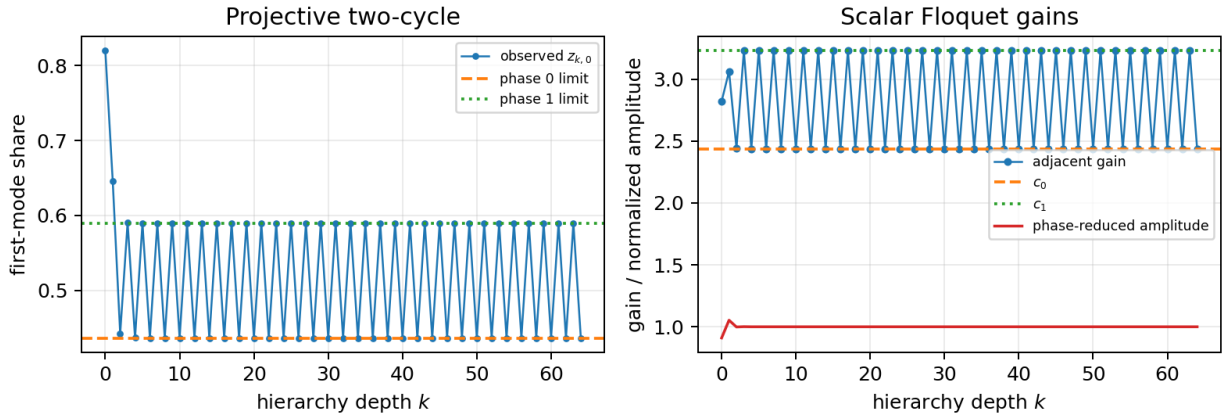
- (a) **Dataset identity.** The exact inputs are `adder.blif`, `arbiter.blif`, `bar.blif`, `div.blif`, `i2c.blif`, `log2.blif`, `multiplier.blif`, `router.blif`, `sqrt.blif`, and `square.blif` under `artifacts/source_netlists/`. File hashes are recorded in the root SHA-256 manifest. Circuit names denote these snapshots, not every version of the corresponding design.
- (b) **Representation.** The BLIF parser creates explicit primary-input, gate-output, and primary-output vertices and one hyperedge for every signal incident on at least two distinct vertices. The implementation constructs a weighted sparse graph projection, sums duplicate CSR entries, and sorts each CSR row by column index before RCM. The same canonicalization is applied to every recursively induced subgraph. This fixes neighbor-visitation order, including degree ties, in the supported software environment.
- (c) **Hierarchy.** Fixed RCM forms one ordering and splits it into 1, 2, 4, $\dots$ nearly equal contiguous blocks until every block has at most eight vertices. Recursive RCM bisects each current module, recomputing its local RCM order above 1,024 vertices and preserving inherited order below that threshold, again stopping at eight vertices.

Autonomous Perron basin and slowly varying envelope



(a) Autonomous projective forgetting.

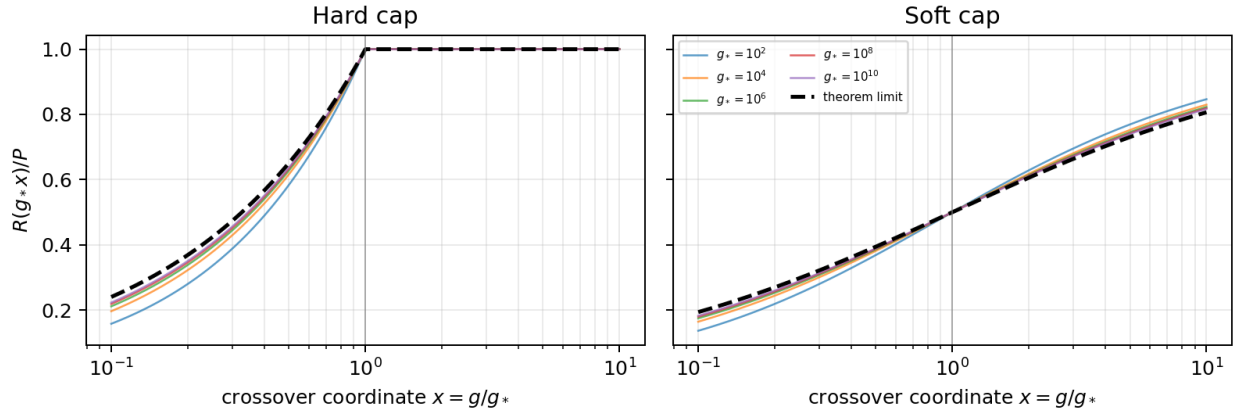
Periodic positive cocycle: one exponent, persistent hierarchy phase



(b) Periodic Perron–Floquet cycle.

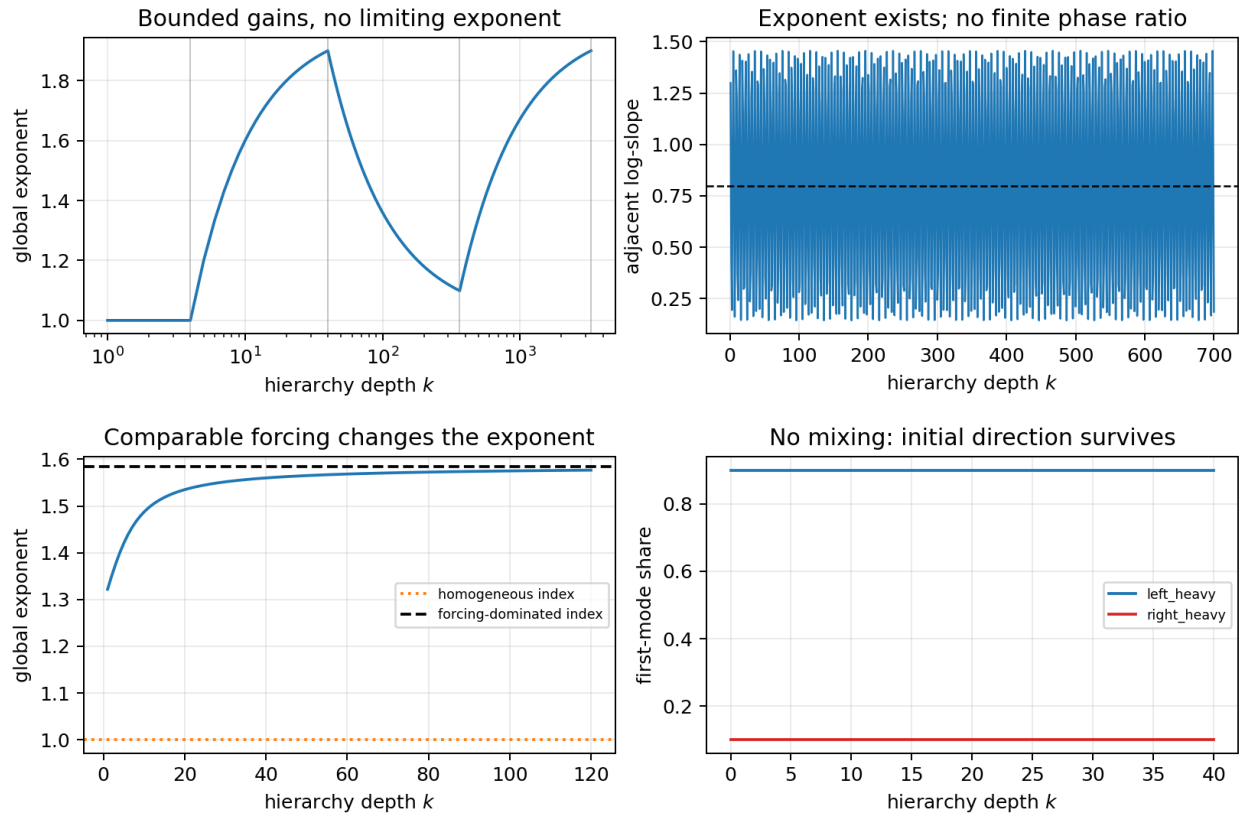
Figure 5: Deterministic checks of autonomous projective forgetting and finite-period phase retention.

Boundary crossover collapse retains the registered cap response



(a) Registered cap responses.

Negative controls delimit the universality basin



(b) Required negative controls.

Figure 6: Deterministic checks of boundary-cap collapse and registered failure modes.

Circuit	Vertices	Fixed-hierarchy $\hat{p}$	Scatter $\hat{p}$
router	374	0.095	0.294
adder	1,405	0.000	-0.029
i2c	1,646	0.807	0.820
barrel shifter	3,599	0.756	0.805
arbiter	12,224	0.976	0.978
square	18,678	0.843	0.818
square root	24,810	0.941	0.936
multiplier	27,318	0.909	0.910
logarithm	32,124	0.864	0.850
divider	57,503	0.947	0.949

Table 2: Diagnostics under the deterministic fixed hierarchy. Values are conditioned on the registered estimator and hierarchy; they are not intrinsic Rent exponents.

- (d) **Traffic and scale window.** A non-I/O hyperedge contributes once to every module it touches whenever it crosses a module boundary. The fanout classes are 2, 3–8, 9–32, and above 32 pins. Unweighted ordinary least squares fits $\log R$ against $\log g$ only on levels with positive internal traffic and

$$16 \leq g \leq G/4.$$

The level-mean estimator uses $\log(\text{mean } T)$; the scatter estimator uses $\text{mean}(\log \max\{T, 1\})$.

- (e) **Mode attribution.** A mode law is fitted only when that mode is positive on at least three retained levels. Profile relative RMSE is

$$\left[\frac{1}{m} \sum_i \left(\frac{\hat{R}_i - R_i}{R_i} \right)^2 \right]^{1/2}.$$

Slope and curvature RMSEs are ordinary root-mean-square differences after second-order finite differences on the log-scale grid.

- (f) **Cross-hierarchy transfer.** The four source-hierarchy mode laws are frozen. One total-amplitude anchor is permitted at the largest common target scale; every other target level is a prediction. Thus transfer cannot refit mode exponents or introduce level-specific corrections.
- (g) **Determinism and records.** The RCM paths invoke no random generator, so there is no empirical seed parameter. With sparse-ordering canonicalization, two independent reruns produced byte-identical CSV, JSON, and PNG outputs. Generated PDFs differ only in creation metadata and are compared by rendered content rather than raw bytes. Environment versions are pinned in `artifacts/requirements.txt`. Executable methods are in `artifacts/epfl_spectrum_analysis.py`, `artifacts/epfl_crg_validation.py`, and `artifacts/cross_hierarchy_mode_test.py`; level tables, summaries, and transfer records are under `artifacts/epfl_results_v2/` and `artifacts/results_epfl_v3/`.

11.2 Hierarchy- and estimator-conditioned scalar fits

The ten fitted values span nearly the whole interval from zero to one under a single registered procedure. This dispersion is descriptive of the retained snapshots and scale windows. It is not a population estimate for circuits in general. The level-mean and scatter estimators differ most strongly on the router, where their fitted slopes are 0.095 and 0.294, respectively, consistent with the finite-level Jensen gap in [Section 10](#).

Hierarchy choice is also material. For the multiplier, the fixed ordering gives $\hat{p} \approx 0.909$, whereas recursive local RCM gives $\hat{p} \approx 0.831$.

Versioned diagnostic note. An earlier measurement contract reported the multiplier pair $0.901 \rightarrow 0.839$. The values $0.909 \rightarrow 0.831$ used here are from the controlling registered protocol, which additionally fixes sparse-row canonicalization, hierarchy thresholds and stopping rules, and the fit window $16 \leq g \leq G/4$. The retained records do not isolate the numerical change to one pipeline modification. The two pairs therefore remain outputs of different versioned measurement contracts; neither is interpreted as an intrinsic hierarchy-independent exponent.

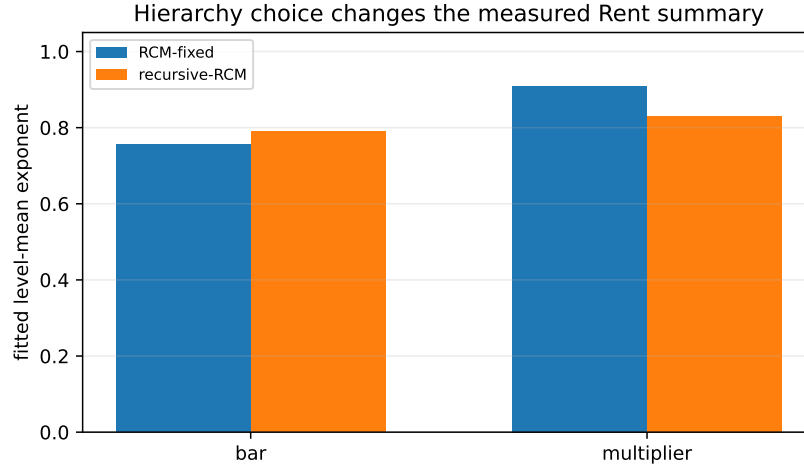


Figure 7: The multiplier scalar fit changes under two deterministic hierarchy instruments even though parser and traffic accounting remain fixed.

11.3 Exact mode identity, finite-scale failure, and transfer

At each retained level, internal incidence is decomposed into the four fanout classes. The arithmetic identity

$$R(g) = \sum_{j=1}^4 R_j(g)$$

holds with zero numerical residual under both multiplier hierarchies. Two scientifically different tests follow.

First, fit one pure law $a_j g^{p_j}$ to each measured fanout mode. Under the fixed hierarchy the fitted mode exponents are approximately

$$(0.931, 0.902, 0.508, 0.231),$$

whereas recursive local RCM gives

$$(0.891, 0.754, 0.696, 0.192).$$

The sum of these global mode fits does not closely reproduce the whole finite-scale curve. Profile relative RMSE is 0.135 under fixed RCM and 0.0478 under recursive RCM. The variance-only pure-mode approximation has slope RMSEs 0.265 and 0.0555, respectively. Hence the real fanout modes are themselves scale dependent; the pure-power specialization of [Theorem 8.1](#) is falsified as a complete finite-scale model.

Second, retain the measured local slopes

$$p_j^{\text{loc}}(g) = \frac{d \log R_j(g)}{d \log g}$$

and their drift. With

$$\omega_j(g) = \frac{R_j(g)}{\sum_k R_k(g)},$$

the exact identities are

$$p_{\text{loc}}(g) = \sum_j \omega_j(g) p_j^{\text{loc}}(g),$$

$$\frac{dp_{\text{loc}}}{d \log g} = \text{Var}_{\omega(g)}(p_j^{\text{loc}}) + \sum_j \omega_j(g) \frac{dp_j^{\text{loc}}}{d \log g}.$$

Using modes measured under the same hierarchy reconstructs the multiplier slope with RMSE 0.00935 under fixed RCM and 0.00318 under recursive RCM; curvature RMSEs are 0.0497 and 0.00876, respectively. This validates the traffic decomposition and numerical implementation of the algebraic identity. Because $R = \sum_j R_j$, the reconstruction is exact; its scientific role is to separate exponent dispersion from within-mode drift.

The cross-hierarchy transfer test is independently falsifiable. Fixed-RCM mode laws transferred to recursive RCM have profile relative RMSE 0.175 and local-slope RMSE 0.0864. In the reverse direction the errors are 0.194 and 0.261. Fanout classes remain identifiable, but their effective scale laws are hierarchy conditioned. A hierarchy is therefore an active part of the measurement rather than a passive coordinate change.

Internal and pad-inclusive profiles also separate at the largest scales, consistent with a terminal-boundary crossover. This observation identifies terminal convention as an active coordinate of the measured profile. A unique cap response Φ is determined only after its response family is registered together with hierarchy, fanout accounting, and estimator.

11.4 Interpretation of the diagnostic

The registered experiment establishes four empirical facts. First, the measurement is deterministic and reproducible in the pinned environment. Second, level-mean and scatter estimators can assign materially different slopes to the same hierarchy. Third, changing the hierarchy construction can change both the scalar fit and the mode laws while leaving parser and traffic accounting unchanged. Fourth, the exact mode-mixture identity closes numerically when its modes and local slopes are measured under the same hierarchy.

These facts locate the diagnostic inside CRG. The BLIF snapshots are logical realizations, so their measured objects are topology- and hierarchy-level profiles. Fixed and recursive RCM are two admissible topology instruments; their separation is a direct measure of hierarchy sensitivity. The profiles belong to one realization and hierarchy at a time, and fixed-hierarchy additivity applies to mode incidence inside that realization.

The pure-mode fit and measured-mode reconstruction answer different questions. Failure of the pure-mode transfer demonstrates that one global power per fanout class does not describe the complete finite-scale profile. Closure of the measured-mode identity verifies the arithmetic decomposition and separates between-mode exponent dispersion from within-mode drift. The dyadic scale grid samples one logarithmic phase; shifted and nondyadic hierarchies supply the additional phase information needed to resolve a periodic factor. The largest scales are terminal affected, so finite-scale mode shares, drift, boundary response, and transfer error are the primary empirical outputs.

11.5 Measurement operators and hierarchy transport

The registered diagnostic can be written as an explicit observation operator. Let H denote a hierarchy construction rule, χ a traffic-accounting projection, W a scale window, and E an estimator. For one fixed legal realization P , define

$$\mathfrak{M}_{H,\chi,W,E}(P) = \left(\{ (g_k, R_k) \}_{k \in W}, \hat{p}_E, \{ R_j, \omega_j, p_j^{\text{loc}} \}_j, \frac{dp_{\text{loc}}}{d \log g} \right). \quad (11.1)$$

The output is not one scalar. It is a versioned tuple containing the measured profile, fitted summary, mode decomposition, local slopes, mode shares, and curvature. The hierarchy and estimator are therefore arguments of the observation map, not metadata appended after measurement.

For two hierarchy instruments H and H' , the same realization can produce

$$\mathfrak{M}_{H,\chi,W,E}(P) \neq \mathfrak{M}_{H',\chi,W,E}(P).$$

This inequality is a property of the measurement contract, not a contradiction. The exact arithmetic identity $R = \sum_j R_j$ is preserved within each instrument, while fitted exponents and global mode laws can change because the modules, interface incidence, and sampled scale trajectory have changed. The multiplier values 0.909 and 0.831 are two coordinates of these two registered observations.

Cross-hierarchy transfer is a stronger test than refitting. Let $\hat{\mathcal{L}}_H = \{ \hat{R}_{j,H} \}_j$ denote the frozen mode laws learned under H , and let $\text{Eval}_{H'}$ evaluate them on the scale sequence generated by H' . The transfer residual

$$\mathcal{E}_{H \rightarrow H'} = \text{Err} \left(\text{Eval}_{H'}(\hat{\mathcal{L}}_H), R_{H'} \right) \quad (11.2)$$

measures whether the source law transports without re-estimating its exponents or introducing level-specific corrections. The bidirectional residuals in the multiplier experiment show that fanout labels remain meaningful while their effective scale laws depend on hierarchy construction.

This formalization places empirical scaling inside the CRG ladder. The realization P belongs to $\mathcal{P}$; its typed incidence profile is extracted by σ_H ; the measurement operator selects a scale projection and estimator; and the resulting scalar or mode law is one observable of that projection. Consequently, comparisons across studies

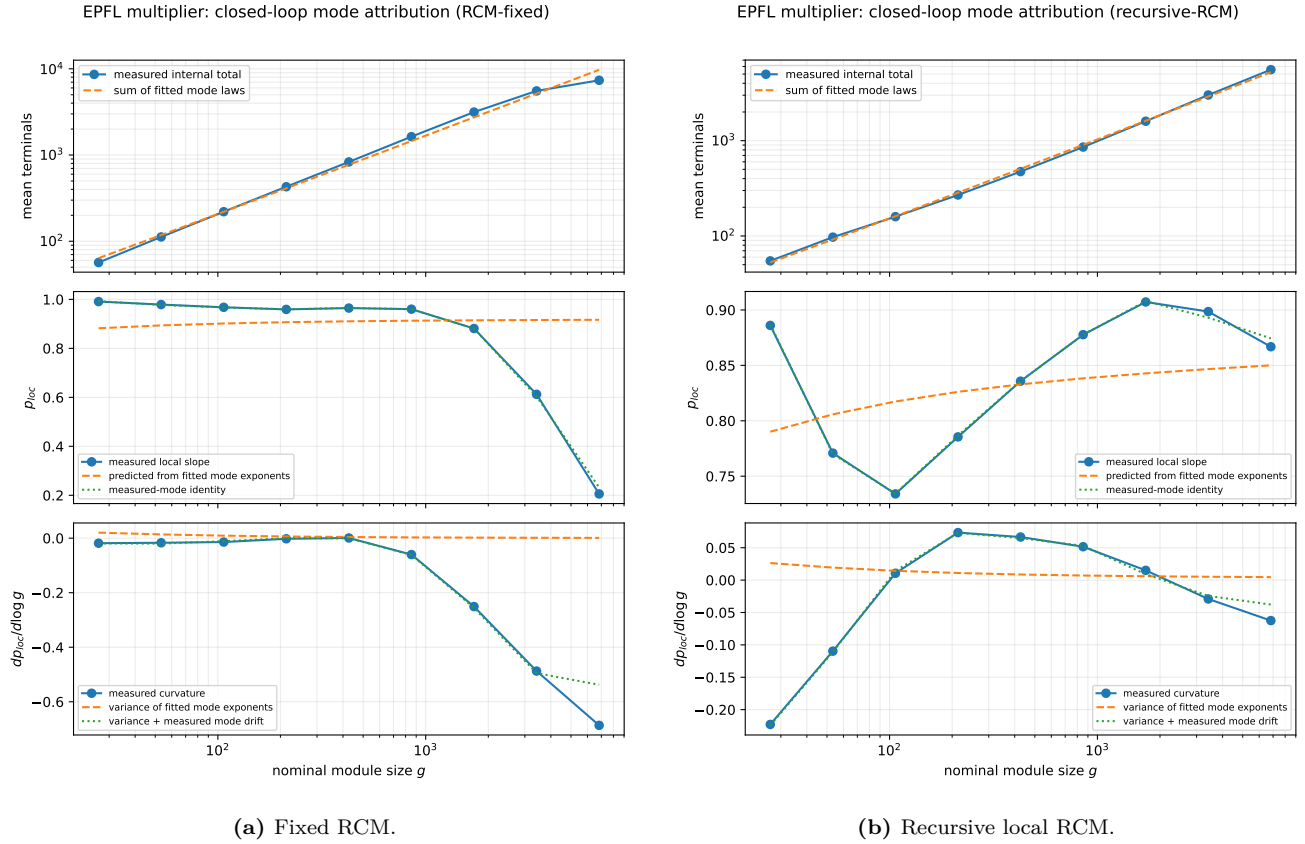


Figure 8: The global pure-mode approximation is incomplete. The measured-mode identity, including within-mode drift, tracks slope and curvature under each hierarchy.

require equality or an explicit transport map for computation, realization, hierarchy, accounting, scale window, and estimator. Once those objects are registered, differences become scientific signals: estimator sensitivity, hierarchy sensitivity, mode drift, phase structure, and terminal response are separately measurable components of the realization geometry.

12 A nonseparable Transformer superblock as a multi-mode profile

The integrated Transformer example combines grouped-query attention, a feed-forward stage, and an optional inter-stage package bridge. Under the declared tensor-work contract,

$$R_{\text{block}}(N) = 4d M_{\text{exec}}(N) + \left[\left(4 + 4 \frac{h_{\text{KV}}}{h} \right) d^2 + 6dd_{\text{ff}} \right] N.$$

The executed-pair function M_{exec} is part of the realization contract: it is N^2 for dense-padded causal execution, $N(N+1)/2$ for ideal causal pairs, and a block-rounded staircase for the default tiled realization. For $h_{\text{KV}}/h = 1/4$, the projection coefficient is $5Nd^2$.

Proposition 12.1 (Transformer multi-mode local exponent). *For $R(N) = aN^2 + \beta N$ with $a, \beta > 0$,*

$$p_{\text{loc}}(N) = \frac{d \log R(N)}{d \log N} = \frac{2aN^2 + \beta N}{aN^2 + \beta N}.$$

The local exponent increases continuously from one to two and equals 3/2 at $aN = \beta$.

Proof. Differentiate $\log(aN^2 + \beta N)$ with respect to $\log N$:

$$N \frac{2aN + \beta}{aN^2 + \beta N} = \frac{2aN^2 + \beta N}{aN^2 + \beta N}.$$

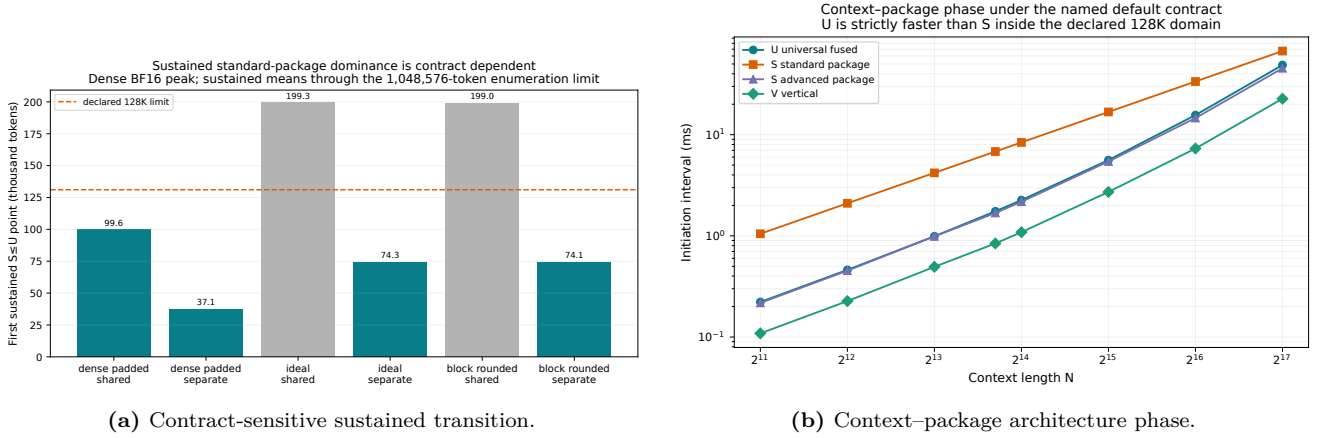


Figure 9: Attention work, feed-forward work, and package-interface traffic form one multi-mode profile. The same crossover structure governs the local scale exponent and the preferred realization.

The derivative of this expression is $a\beta N/(aN + \beta)^2 > 0$, and the endpoint limits follow directly. $\square$

Under disjoint single ownership, the steady-state bridge satisfies

$$R_{\text{bridge}}(N) = 2Ndb_{\text{elem}},$$

an exponent-one mode. Its fraction of a quadratic-dominant total decreases with context, while its absolute byte count continues to grow. Package bandwidth, fast-state feasibility, and execution coefficients therefore decide whether splitting is useful at finite scale.

Under the named default contract, the universal realization remains faster than the standard-package split realization throughout the declared $N \leq 131,072$ range. Across the larger finite enumeration the split class first reaches the universal class near 199 thousand tokens, with a short block-rounding reversal before sustained dominance. Alternative pair-count, peak, and package-accounting contracts move the transition from approximately 37 thousand to 449 thousand tokens. The crossover is a contract-indexed phase boundary, not a workload-intrinsic context length.

The example connects the scaling theory to architecture selection. A statically fused realization can be state-infeasible, a time-shared universal realization can restore feasibility with an efficiency cost, and a split realization can restore specialization while activating the exponent-one bridge. The relevant object is the complete mode geometry and its technology observation.

13 Conclusion

The generalized law changes the scientific status of a Rent-like exponent. A measured exponent is one conditioned observation of one legal realization, not a resource-free scalar of the computation. Exact powers, slowly varying corrections, hierarchy phase, mode crossover, interface reach, terminal saturation, and estimator effects occupy distinct classes with distinct extrapolations.

The theory supplies both output characterizations and generative mechanisms. Stable dilation limits yield regular variation. Phasewise ratio limits characterize finite hierarchy cycles. Positive Perron–Floquet dynamics produce one sufficient mechanism for those cycles. Arithmetic composition turns heterogeneous modes into exact slope and curvature identities. Cap response and dense-anchor collapse connect finite terminal behavior back to the interior scaling class.

The public-netlist results instantiate this calculus. Under the registered protocol, the multiplier has fitted level-mean slopes near 0.909 and 0.831 under two deterministic hierarchies. Measured modes close the arithmetic identity inside each hierarchy, while pure-mode approximation and cross-hierarchy transfer expose the conditioning of the fitted law. The Transformer profile extends the same analysis to a multi-mode computation in which scaling and technology jointly determine the preferred realization.

The result is a generative, hierarchy-aware theory of multiscale communication from computation-realization geometry.

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